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Verification of Investment Opportunities on the Cryptocurrency Market within the Markowitz Framework

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Abstract: The aim of the paper is to reveal if the classical approach for asset allocation can be reflected on an innovative market of cryptocurrencies. Markowitz rebalanced portfolio technique is employed for this purpose. The filtering of coins for optimization is done on the whole scope of cryptocurrencies available for the time horizon of the study and only 52 coins get into the portfolio at least once. There are four primary strategies produced within a set of assumed optimization parameters together with four benchmarks for each. The benchmarks are Bitcoin buy-and-hold, S&P500 buy-and-hold, equally weighted portfolio and portfolio weighted by market capitalization. While looking at the performance measures, it is concluded that Markowitz strategies outperform their benchmarks for every set of parameters. The results of the sensitivity analysis suggest that there is a big potential in finding profitable strategy of investment on cryptocurrencies. The change of parameters: look-back period, rebalancing window, transaction cost as well as optimization objective impact the strategies performance significantly. Eventually, there appear a strategy in the sensitivity analysis, which performs better than the primary ones due to the prolonged parameters of look-back period and rebalancing window.

Keywords: Portfolio analysis, Markowitz framework, cryptocurrencies, investment strategies, asset allocation

JEL codes: C20, C22, C61, C80, G14, G17

1. Introduction

The cryptocurrency inception gave a rise to the era of decentralized digital peer-to-peer network of virtual currencies, which continues to grow, develop and evaluate for around 10 years now. Bitcoin was the first established coin, proposed by Nakamoto (2009), which caused a huge interest among researchers and quite quickly gained a widespread acceptance. Bitcoin's foundation and increased number of its transactions caused appearance of other cryptocurrencies, which are designed within the similar cryptographic technology called blockchain and are being "mined" through solving various mathematical algorithms. The motivation behind creation digital coins is different: traditional money system's improvement, Bitcoin's disadvantages addressing, raising capital through ICO, transfer of value and many others. However, all existing coins have a common characteristic, namely they can be considered as innovative investment opportunity (Lee et al. 2017).

The scope of cryptocurrencies continues to increase as time passes by, together with a number of unconventional hypotheses about pros and cons of such digital invention. There is an ongoing discussion being conducted with regards to the cryptocurrency's characteristics, in particular, addressing whether the digital currencies can be considered as a new class of assets with the opportunity of an anticipated return and whether their properties can be compared to the traditional assets (Glaser et al. 2014, Baek and Elbeck 2015, Kosc et al. 2019). Most researches still focus on the sole Bitcoin examination, which occupies the major part of the crypto market in terms of market capitalization, as well as, on its comovement with world-wide indices, currencies, gold, bonds and equities (Ji et al. 2017, Brandvold et al. 2015, Dyhrberg 2015, Bariviera et al 2017). However, in order to be able to make any sensible conclusions about the digital market investment opportunities, instead of concentrating solely on Bitcoin, it is essential to examine multiple number of coins, their behavior and correlation with each other within the market through time.

Digital coins are very volatile, which is proven when having a closer look at aggregated statistics of annualized return and annualized standard deviation for a number of cryptocurrencies. On the one hand, such characteristics imply a huge risk of potential loss and quite low power of predictability, which already makes the investment on this market be ambiguous and controversial. But, on the other hand, under these circumstances an investor may wonder if such abnormal properties resulting from huge price fluctuation could create an opportunity of gaining an excess return consistent with the level of volatility.

A breakthrough theory regarding portfolio optimization proposed by Markowitz (1952) has been verified for the huge range of the classical assets. It is considered as one of the successful techniques of asset allocation and finding efficient portfolio at the optimal risk-reward level and is widely utilized across traditional financial markets. Application of this methodology results in profit for various number of regulated assets, which means that it can be also checked on the new class, namely cryptocurrencies. Nowadays, many digital coins can be treated as credible and liquid for the investment as well as the technology behind them, is being highly developed. The combination of emerging class of assets with already inspected technique may have a potential for brighter investment future

The purpose of this research is to examine the behavior of the cryptocurrency market from the point of view of an investor willing to verify the power of a new asset class and watch if the conventional foundation regarding investment techniques could be successfully replicated on the innovative market. In order to check such hypothesis, a portfolio of cryptocurrencies is initiated and rebalanced through time. Even though we bear in mind the potential inefficiencies of hardly investigated digital market, we utilize the classical approach widely used for standard assets, namely, the Markowitz framework, in order to construct a portfolio of cryptocurrencies on the basis of some predefined filtering criteria, which will be explained later in this paper:

1. The primary goal is to see whether such digital portfolio, defined within Markowitz (1952) technique for traditional diversified portfolios, can generate a profit and how it can be compared to the benchmarks, which represent several ways of passive investment. Equity lines obtained in the process of rebalancing will eventually reveal if the digital-coin portfolio can be efficient.
2. The research will be focused on gathering and applying several combinations of optimization parameters and examining the resulted strategies within a number of statistical measures.
3. Two optimization objectives will be employed: maximizing Information Ratio (IR) and minimizing variance.
4. Very important foundation would be to discover which set of assumed optimization parameters produce the best results in terms of strategy attractiveness in terms of return-risk ratio.
5. Another aim of this work would be to examine how transaction costs increase may affect the strategies’ performance.

The analysis conducted in this study has a composite character and together with a primary aim of verifying Markowitz technique on the new asset class, asks several concomitant questions.

This paper is structured the following way: in the first section we will review the literature regarding the digital market foundations and cryptocurrencies portfolio construction, in the second section the methodology of filtering criteria and portfolio construction for the research will be described, the third section will reveal the results and the sensitivity analysis, in the last section the conclusions will be made.

2. Literature review

Markowitz technique of portfolio optimization is widely applied for traditional assets. Many papers were written suggesting that the investment within this technique is usually more profitable than naive asset allocation or buy-and-hold strategies due to the mechanism which searches for diversification opportunities and maximizing risk-adjusted return. For instance, the studies which apply Modern Portfolio Theory (MPT) address the following topics: classical Markowitz on equities (Ivanova et al. 2017), examining the technique (Bai et al. 2016, Keller et al. 2014, Keller 2015), optimization with alternative criteria (Sakowski et al. 2016), comparison with other investment techniques (Bessler et al. 2012, Becker 2009) and many others.

There become more and more articles and papers published within the topic of crypto market, regarding legal, technical, financial, social aspects. However, the most meaningful for this study is the matter of two research areas: digital coins investment properties and portfolio performance with cryptocurrencies. There are several questions being asked by researchers nowadays regarding cryptocurrency market. The one about, what drives the prices of Bitcoin and other digital currencies, is still very disputable, since there is no single answer. Quite a number of authors conclude that the influence of global macroeconomic indicators on Bitcoin's price fluctuation is insignificant (Brière et al. 2015, Polasik et al. 2015, Ciaian et al. 2016, Bartos 2015). It is also widely discussed that the price of Bitcoin is driven by rather non-financial indicators, namely Bitcoin attractiveness indicators (Ciaian et al. 2016, Kristoufek 2013), attention in the media (Garcia et al. 2014), anonymity of Bitcoin payment transactions (EBA 2014, Yermack 2013), computer programming enthusiastic promotion (Yelowitz and Wilson 2015), speculative bubbles (Cheung et al. 2015, Cheah and Fry 2015), and the cost of mining Bitcoin (Garcia et al. 2014).

Many authors also concentrate their attention not only on the nature of prices, but on investment characteristics of digital coins, in particular, the relation of Bitcoin with popular financial assets. Several papers address the presence of dependencies between Bitcoin and classical financial instruments: Dyhrberg (2016) examines the dependencies between Bitcoin and UK equities as well as exchange rates EUR/USD GBP/USD; Baur et al. (2015) studies the relation between Bitcoin and valuable metals, currencies; energy commodities discussed in Bouri et al. (2017).

Since it is discovered that the Bitcoin's price is hardly affected by global economic variables, its behavior in the portfolio is then considered in terms of diversification opportunities. Briere et al. (2013) called digital coins as alternative investment with diversification benefits, when investigated the consequence of adding Bitcoin to the portfolio of wide range of standard assets. Eisl et al. (2015) adds Bitcoin to an equity portfolio. Trimborn, Li, and Härdle (2017) support this hypothesis, by saying that cryptocurrencies diversify well the mainstream assets. Bjordal and Opdahl (2017) and Henningsson (2019) also claim that adding Bitcoin to a portfolio of standard assets improves its performance in terms of Sharpe ratio.

Nadarajah and Chu (2017) and Tiwari et al. (2018) find the cryptocurrencies to be efficient in terms of predictability of returns and Bartos (2015) studies the informative efficiency of cryptocurrency market. Moreover, Bitcoin and other cryptocurrencies are protected from inflation originating from national government changes or restrictions, since they are decentralized and are not legislated by any government institution (Magro, 2016). This creates a "safe haven" for investors and make cryptocurrency look attractive. Dyhrberg 2015 addresses the hedging capabilities of Bitcoin and locates it in the middle between gold and US dollar. However, Bouri et al. (2017) question such hypothesis. Using dynamic conditional correlation model of Engle (2002), authors find that Bitcoin is proved to be a good diversifier but can be considered as a hedge and safe heaven only in some rare cases. The same is proved by Klabbers (2017).

Despite the fact that many relations of Bitcoin are studied, other cryptocurrencies features are overlooked, even those, which are relatively large in terms of market capitalization. Quite modest amount of research has been devoted to other-than-Bitcoin currencies, which seems undeserved. Since the cryptocurrencies are to be considered as a separate asset class, the characteristics and the potential of this class need to be studied, which would involve some common features. If nothing common could be revealed, then each cryptocurrency separately can be named as alternative investment but not a member of a new digitalized asset class.

The literature issued so far regarding portfolio consisted only of cryptocurrencies is very scarce. Brauneis and Mestel (2018) were ones of the first proceeded with an examination of how sole cryptocurrency investment would behave in the risk-return framework found by Markowitz (1952). The number of coins in study was 1096, the whole crypto market for the moment of data downloading. There were created 24 rebalanced portfolios and the conclusion regarding the investment was made. According to this study the naïve investment strategy outperforms portfolio-optimized return. Platanakis et al. (2018) also undertook the trial of portfolio establishment within mean-variance optimization, considering only 4 assets. The findings imply that in their studies there is no much difference between naive and optimized investment results. Another substantial finding is described by Corbet et al. (2018), who suggests that having studied wider range of cryptocurrencies, low correlation is detected between particular coins on the crypto market. This study also allows investors to make use of diversification properties of cryptocurrencies within their class. Liu (2018) establishes 6 portfolio selection models, employing 10 cryptocurrencies and concludes that naive 1/N portfolio's Sharpe ratio cannot be reached by any of other models. Anyfantaki et al. (2018) also investigate the diversified portfolio opportunities of a number of cryptocurrencies and come up with a conclusion about improvement while adding digital coins to standard asset portfolio. Similar conclusions made by Henningsson (2019).

Despite many discoveries regarding promising investment opportunities on the innovative market, there is a number of drawbacks regarding the investment on cryptos. One significant in the operational risk of digital stock exchanges, some of which collapsed. Exchanges like Mt Gox, Flexcoin and Sheep Marketplace were hacked and implied the disappearance of Bitcoins value of hundreds of millions of dollars[1].

[1] Source: Managing the risks of cryptocurrency - BAE systems 2016.

3. Methodology

3.1 Markowitz portfolio theory

Modern Portfolio Theory designed by Markowitz in (1952) changed the way investors look at asset allocation. The key concept, which makes MPT revolutionary and so much applicable is the fact that Markowitz found the existence of trade-off between market risk and expected return of the portfolio. This, in fact, means that for the risk-averse investor it is possible to maximize the return of his portfolio with respect to the certain level of risk. Markowitz also described the rationality of investors in constructing optimal portfolios under conditions of uncertainty with the use of statistical measures for expectation and variance of return. Another substantial foundation was the fact that not only types of assets you include into your portfolio, matter most of all, but rather the weights you devote to these assets. This, in fact, means that investors should not consider only the nature of particular assets and their sole returns, but have a look at the relation and dependencies between them and figure out how these assets perform being put together into one basket. It brings us to the concept of diversification, which is considered to be one of the ways of risk reduction and, consequently, portfolio performance improvement (Fabozzi, Gupta, & Markowitz, 2002). Then, the natural question arises, namely, how to diversify portfolio successfully. The answer was also provided by MPT. Diversification solution is contained in the statistical concept, which is called correlation. The less correlation is found between two or more assets, the higher the diversification possibilities for a portfolio, and, consequently, the lower the risk bared by the investor. The wider theoretical background on the methodology of Markowitz optimization can be found in the handbooks by Elton et al. (2014) and Markowitz et al. (2013)

Another significant point mentioned by Markowitz is the statistical distribution of the returns. In this paper this is a very questionable issue because, while considering cryptocurrencies market, it is apparently, not the case where the returns distribution is normal. If to refer to the previous studies, most authors argue this is truly necessary, however, there are some like Levy & Markowitz (1979), who suggest that normality is not an essential assumption and non-normal returns can be also taken into account. Hence, in this research we should not expect any significantly undesirable results due to non-normality of cryptocurrencies distribution, because even on standard markets the non-normality of instruments exists. The excess kurtosis for the digital coins could be though higher than generally for the classical assets.

3.2 Data analysis and filtering

3.2.1 Data description

The purpose of the research in this paper is to employ all cryptocurrencies available on the market and choose the coins for portfolio optimization, being rebalanced through time and along the crypto market changes. Hence, the data of daily observations for 2140 cryptocurrencies were downloaded as of 26th April 2019 from coinmarketcap.com**,** which contains the values of all cryptocurrencies from the inception of Bitcoin till that date. The IT tool used for this research is R and the data is downloaded with the help of one of its packages called "crypto". The function `crypto_history()` scraps the data for the whole crypto market from coinmarketcap.com. The dataset used for the research contains four columns with values "date", "name", "close", "market", where "name" is the cryptocurrency name indicator, "close" is the close price of digital coins and "market" is the market capitalization.

The downloaded data has daily frequency and takes into consideration also the weekends, due to the fact that the cryptocurrencies are traded through the whole year. Since the number of new cryptocurrencies' appearing on the market is relatively large, many cryptos possess scarce historical observations due two reasons: either they appeared recently, or they ceased to exist at some point in the data range considered in this study. Hence, these coins cannot be employed within Markowitz framework (described in detail in Section 3). As a consequence, cryptos with less than 60 days of observations were deleted from the dataset, because it would have directly impacted the variance-covariance matrix estimation. However, the fact that all coins with more than 60-days observations take part in the filtering process (described in Section 2.2.2.), we can avoid the self-selection bias and not improve the results of the filtering artificially. Moreover, the coins with few observations could still have a potential to get into the portfolio depending on the filtering criteria, especially if they have sudden huge price increase (while pursuing the objective of Information Ratio maximization). These crypto may disturb the portfolio due to low liquidity and significant inconsistency. Hence, a further filtering method should be discovered and applied.

The start date of the data for this research was restricted to begin from 2014-01-01 and end up on 2019-04-26 (in Section 3 the time horizon will be described) The whole number of observations then constitutes 1941 days. Such action is undertaken in order to have sensible number of cryptocurrencies in the beginning of the data set and be able to have enough pool of coins to choose from, especially taking into consideration that at each rebalancing date the scope of cryptos in the portfolio is going to change. More than five years of prices is taken under consideration in this research, which may allow to see how the market evolves and develops as well as if there is a trend in the profit opportunities. The dataset is checked for missing values and flaws and turns out to be ready for the study. The filtering criteria, which allow to choose particular coins into the portfolio for every rebalancing date is addressed and described in the next section.

Analysis of such a huge number of assets looks complicated. Even after deleting short-history coins, we are left with quite significant amount of time series of the digital assets. Since the share of Bitcoin's market capitalization as well as its price are very high in comparison to other coins and the overall digital market performance follows the Bitcoin's trend, we do not analyze aggregated statistics of the coins in time, because it would not bring sensible conclusions regarding its behavior. On the other hand, one-by-one currency analysis would also be inefficient and would not allow to get a full picture of the crypto market either. Hence, in order to provide an overview on the data utilized in the study, the data analysis will be presented within the section 2.2.2, which describes the methodology of choosing virtual coins for the optimization.

Index S&P500 is chosen to be one of the benchmarks[2]. The daily data of close prices is downloaded from stooq.com for the same range under consideration, namely from 2014-01-01 to 2019-04-26. Since the index is traded 252 days per year due to the reason that, opposite to cryptocurrencies, weekends are non-tradable days, the number of observations for this time series constitutes 1338.

3.2.2 Filtering criteria

The key intention of this research is to find out whether the investment in cryptocurrencies can be profitable within traditional way of investment thinking. Several points then should be taken into account.

First of all, usually, investors prefer to choose financial instrument either with higher return or lower standard deviation. It also regards the willingness to consider the trade-off between these two characteristics. Simple daily returns for the last month as well as rolling standard deviation with the window 30 days were calculated using the close prices for further analysis. While referring to the cryptocurrencies market, it is concluded that the returns as well as the standard deviation are abnormally huge when compared to classical assets. This implies the fact that it is necessary to check how each of these conditions work in the filtering process.

[2] S&P500 as well as other benchmarks for the study are described in Section 2.3.2.

Secondly, the instruments in the diversified portfolio must possess liquidity, especially in the situation when the portfolio is being rebalanced, which implies selling and buying new assets at certain price. Again, a major number of digital coins have very low market capitalization, which can be inconsistent with their high once-in-a-while return, hence it is important to control over this while choosing assets for optimization. In order to do this, the rank is put for the cryptocurrencies at every day of observations and it is worth to mention that it changes from date to date with the new coins appearing on the digital market.

In order to get to Markowitz portfolio optimization, it is essential to prepare the pool of cryptocurrencies, among which the weights will be distributed at every date of optimization. The clean data without short cryptos containing less than 60 days is filtered in three methods in order to obtain the best set of cryptocurrencies for the digital portfolio.

Filtering Method I

The initial assumption of the research was to emphasize the importance of market capitalization role in the coin's attractiveness but at the same time give a chance to smaller coins to get into portfolio if their performance is sufficient.

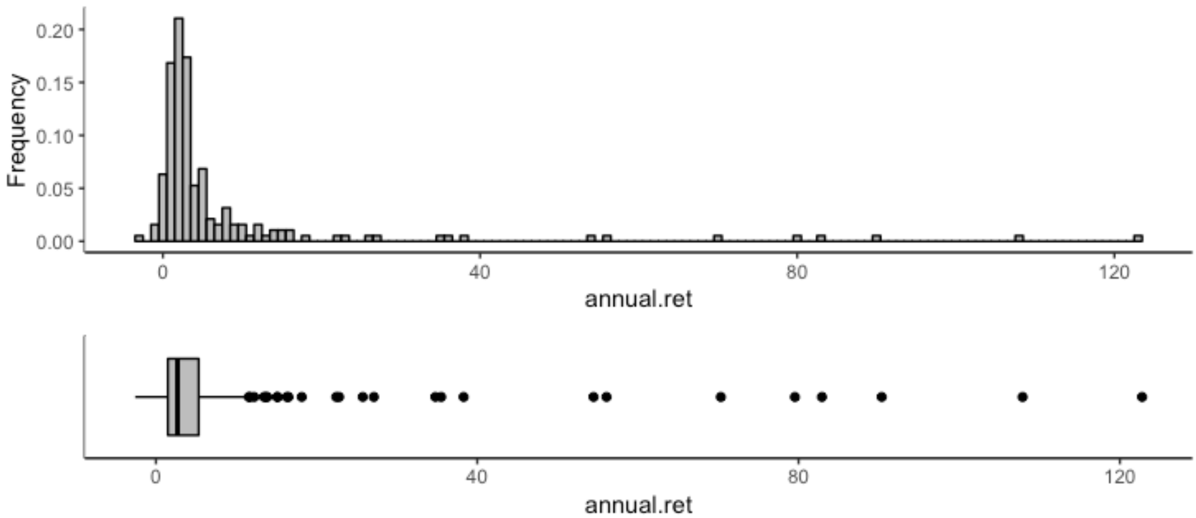
Hence, it was decided to filter 100 highest market capitalization coins for each rebalancing date. The second step of the initial filtering assumption was to utilize the momentum effect for those 100 and choose 10 coins with the highest return for the last month and the lowest standard deviation for the last month. Those two groups would be compared in terms of the resulting strategies. The described method is named as the Filtering Method I. Cryptocurrencies, those major ones in particular, have shown substantial increase in prices through time, which suggests that there is an opportunity of obtaining profit from momentum investment. The number of assets for the portfolio was chosen to be 10, because, it could allow for the diversification needs, on the one hand, but would not imply too much dispersion of weights, on the other.

The data analysis for chosen coins was conducted and it turned out that many groundless cryptocurrencies get into the pull for optimization. The coins, which experienced significant increase in price once in their history showed good last-month return a certain point in time, but, in fact, the return from such coins in the portfolio would be negligible or the suddenly raised market capitalization still would not allow to buy the coin without impacting the market price.

Filtering Method II

The above described foundation made us to cease the range of the first filtering step from 100 highest market capitalization coins to 30. According to the rational assumption this would eliminate the low capitalization coins and restrict the sudden peaks in the price series at the step of applying momentum effect. So, at this point, two groups of coins are considered: 10 cryptos with highest return for the last month chosen from 30 and 10 cryptos with lowest standard deviation for the last month chosen from 30. Such method represents the Filtering Method II. The number of coins, which would take part in the optimization within this method of filtering, at least in one rebalancing (RB) period, taking into consideration both groups is 190. The data analysis then was conducted and the histogram and boxplot of annualized returns for these coins was obtained in order to see how the return of the cryptocurrencies filtered along the assumed criteria for the optimization are distributed. The results are presented in Figure 1.

Figure 1. Frequency of annualized returns (%) for 190 coins filtered for the Markowitz optimization within Filtering Method II for the period 2014/03/02 – 2019/04/26.



Source: own calculations based on period 2014/03/02 – 2019/04/26. Filtering Method II incorporates two steps: choosing 30 coins with highest market capitalization and then filtering 10 coins with the highest last-month return for every rebalancing date. Annualized returns of all coins which take part in optimization through the time horizon (190) are calculated and their frequency is shown on histogram and boxplot.

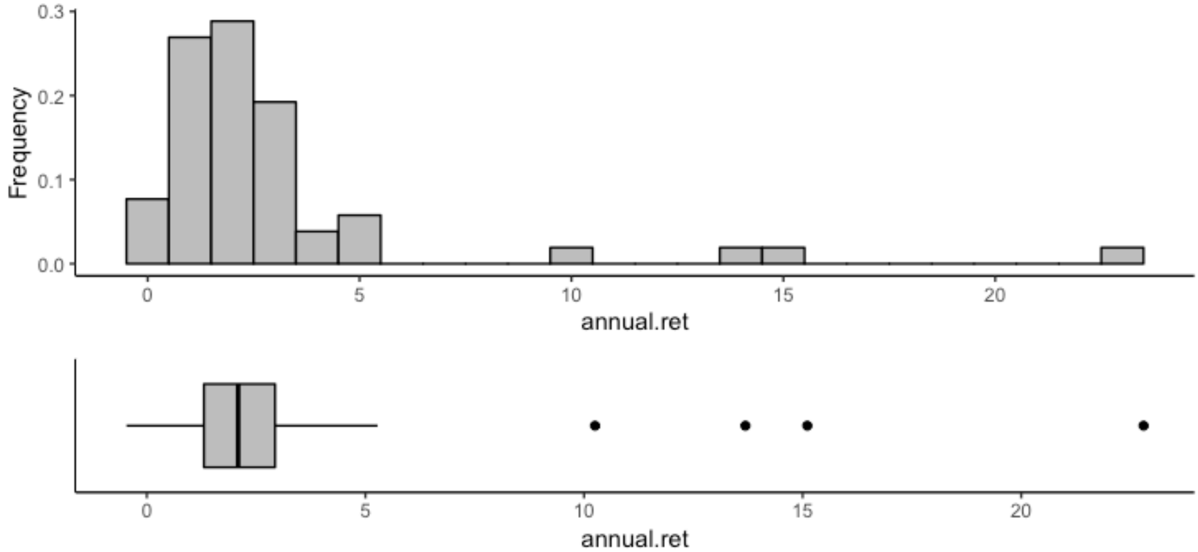
The restriction to 30 highest market capitalization coins still overlooks some low capitalization coins with insignificant price. Strongly right skewed histogram shows that the annualized return of several coins is inconsistently huge. It is easily observed in the Figure 1, that there is a lot of outliers among coins, which are supposed to take part in optimization. NewYorkCoin, the farthest outlier, has an annualized return of 122.76%, however its price at the moment of huge peak was slightly more than \$0.001 and then dropped again together with the market capitalization going down to slightly more than a million dollars. The similar case is with SounDAC, empower coin, Dimecoin and other strongly deviating outliers. The above presented analysis supports the idea of recalling the Filtering Method II from the research.

Filtering Method III

In order to make the portfolio more sensible and consistent with the investor needs and assumptions, the Filtering Method III was created. It was decided to resign from employing the momentum effect on the basis of monthly return and standard deviation for filtering. Hence, the only filtering aspect in the third method is applying the ranking of highest market capitalization for every rebalancing date and choosing first ten cryptos to the portfolio. It ensures that only the major coins get into the optimization each time the portfolio is rebalanced. The number of coins chosen to participate in the optimization now ceased to 52. Data analysis was conducted the same way as for Filtering Method II and Figure 2 presents the resulted histogram and boxplot for the filtering technique which takes into consideration only the market capitalization as a parameter for choosing cryptos to invest.

The histogram shows that most of the annualized return are concentrated between approximately -0.5 and 5.5, representing the positive and adequate values for most coins, however, there are still some extreme values. Fedoracoin, NuShares, Pluton, Paycoin are the outliers on Figure 2, which went through filtering due to the sudden once-in-a-history increases of market capitalization. Such peaks imply significantly higher return on the annual scale. Though, the prices of those 4 crypto hardly reach $1 currently, hence, their participation in the optimization for sure will not be continuous due to rebalancing. Since cryptocurrency market is very volatile, such high values of annualized returns are assumed to be still acceptable for the current research.

Figure 2. Frequency of annualized returns (%) for 52 coins filtered for the Markowitz optimization within Filtering Method III for the period 2014/03/02 – 2019/04/26.

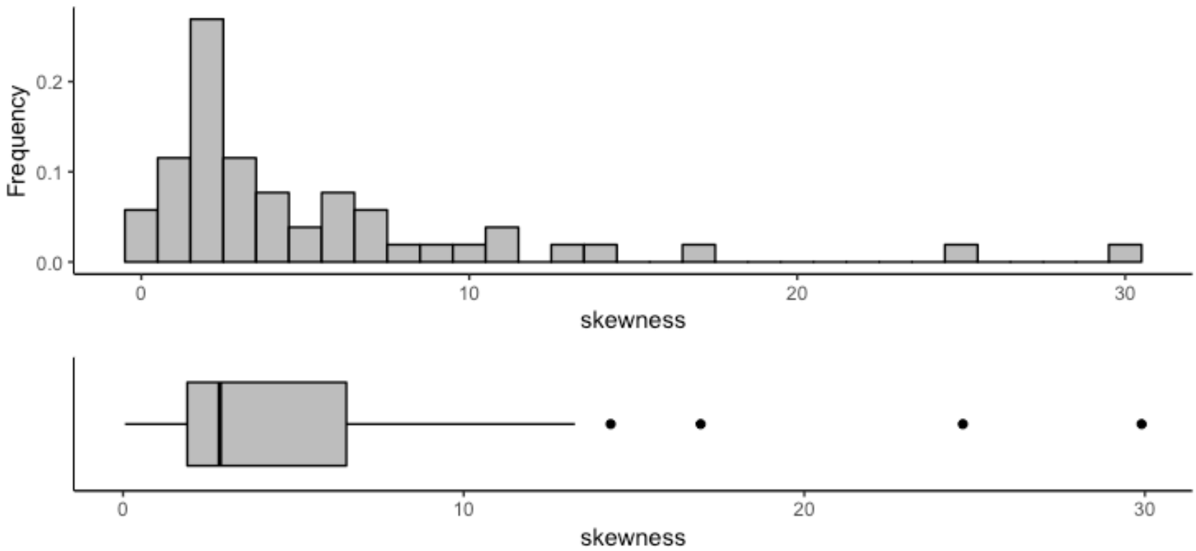


Source: own calculations based on period 2014/03/02 – 2019/04/26. Filtering Method III implies choosing 10 coins with the highest market capitalization for every rebalancing date. Annualized returns of all coins which take part in optimization through the time horizon (52) are calculated and their frequency is shown on histogram and boxplot.

The annualized returns of the coins chosen according to the Filtering Method III are also studied in terms of their skewness and kurtosis. The values of these measures turn out to be quite high as well. The set of coins for optimization is strongly right-skewed, which implies that data for the optimization is highly unsymmetrical and is not comparable to the standard normal distribution. The skewness distribution for 52 coins chosen for the optimization is shown on the Figure 3.

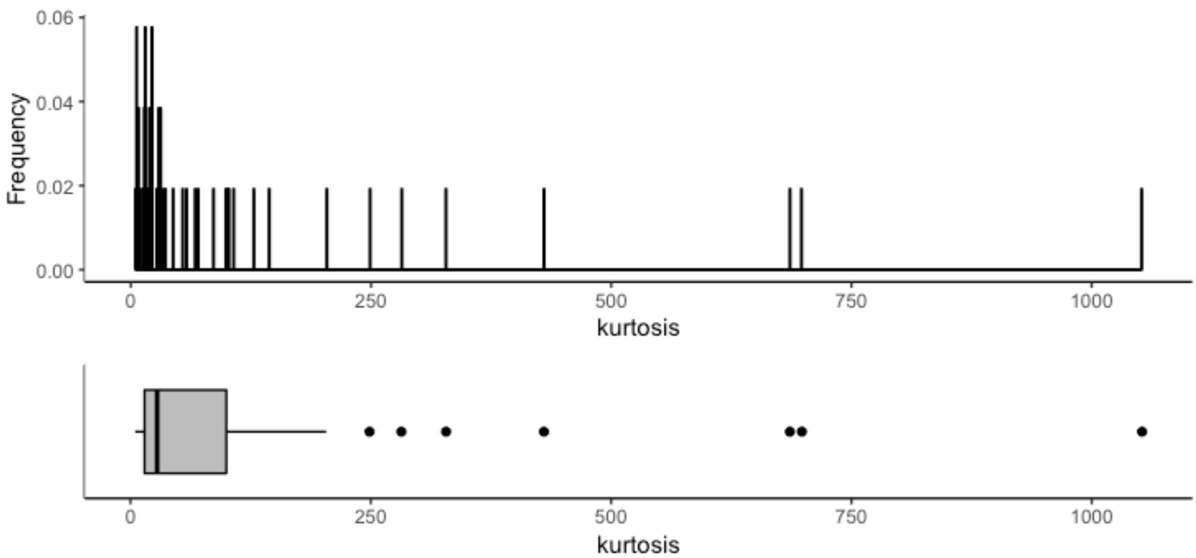
If referring to kurtosis coefficients, the histogram of their distribution looks even more disturbed. The number of outliers is quite high and the values themselves are huge. Figure 4 suggests that there is no negative kurtosis, however the value, for example, for NuShares reach the value of 1052 due to its unrealistic variation in price and presence of various outliers. Quite a significant number of digital coins possess kurtosis higher than 100 in the set under examination, which implies huge leptokurtic characteristics for cryptocurrencies market. Elender et al (2017) also addresses the abnormal leptokurtic characteristics of digital coins. These observations can actually support the idea of excess kurtosis of cryptocurrencies relative to the classical markets.

Figure 3. Frequency of skewness coefficients for 52 coins chosen for the Markowitz optimization within Filtering Method III for the period 2014/03/02 – 2019/04/26.



Source: own calculations based on period 2014/03/02 – 2019/04/26. The frequency of skewness coefficients for 52 coins filtered in accordance with the highest market capitalization for Markowitz optimization.

Figure 4. Frequency of kurtosis coefficients for 52 coins chosen for the Markowitz optimization within Filtering Method III for the period 2014/03/02 – 2019/04/26.

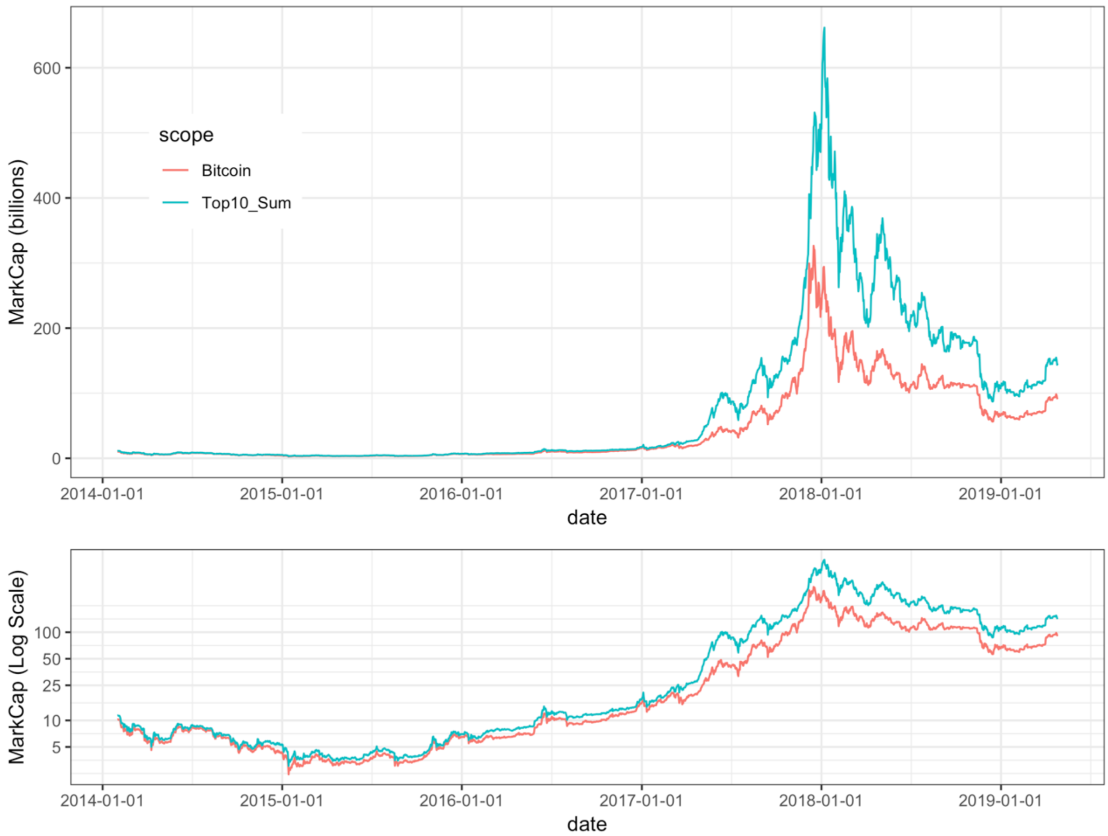


Source: own calculations based on period 2014/03/02 – 2019/04/26. The frequency of kurtosis coefficients for 52 coins filtered in accordance with the highest market capitalization for Markowitz optimization.

While having such filtering, several observations can be also made even before applying Markowitz framework for strategies creation. First, Bitcoin will always take part in the portfolio optimization, due to the fact that it has been occupying the major part of the cryptocurrencies market since its foundation. Secondly, a curious fact is that during producing trial strategies within Filtering Method II, it was noticed that Bitcoin actually participated in top 10 coins for optimization quite rarely despite its high price and market capitalization. It also implies that many coins in the short term could outperform Bitcoin in terms of their return, but they are not chosen to the optimization due to low liquidity.

The next observation is the fact that the sum of market capitalization of ten chosen cryptos should follow the trend of Bitcoin's market capitalization due to its major share on the crypto market. Figure 5 supports this theory and presents the performance of two time series.

Figure 5. Bitcoin's market capitalization compared to the sum of top 10 highest market capitalizations (in billions).



Source: own calculations based on period 2014/03/02 – 2019/04/26. First panel presents the market capitalization of Bitcoin and the sum of ten chosen coins' market capitalization. Second panel shows the same two time series but on the logarithmic scale in order to observe the changes during the time range of the study.

Having a look both at the linear plot and at the logarithmic scale, we can see that two lines behave very similar and the cryptocurrencies leader Bitcoin prevails in the top-ten sum along the whole-time range. Since second quarter of year 2017 the sum of top ten market capitalizations has higher values, which suggests that market capitalization of other-than-Bitcoin cryptocurrencies increased significantly since then. Hence, the intervals where two lines converge to each other imply the relatively diminishing market capitalization of assets on the crypto market compared to Bitcoin. The second panel of Figure 5 presenting the market capitalization on the logarithmic scale supports this idea.

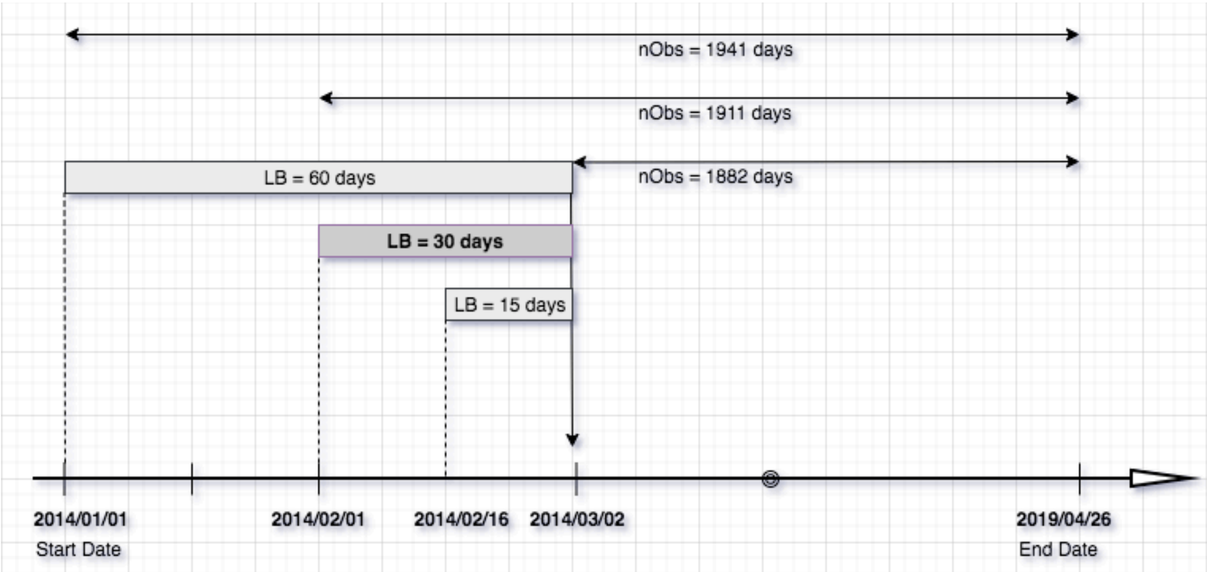
It is also worth to mention is that the concerns of liquidity and adequacy of cryptocurrencies for optimization are generally resolved within Filtering Method III. Only several coins still deviate and show abnormal statistical values, like high annualized return or huge kurtosis. However, these findings also support the idea of cryptocurrencies being very innovative and non-usual way of investment. Liquidity is ensured to exist for every rebalancing date since the aim of the filtering was to choose only credible coins with highest market capitalization (Filtering Method III). Market capitalization is assumed to be the key factor of coin's attractiveness for the portfolio in this study.

3.3 Investment strategies definition

3.3.1 Markowitz portfolio construction

Despite the fact that the data for the research starts on 2014-01-01, at the day of portfolio construction it is necessary to have observations for past 60 days for the purpose of having historical data for Markowitz optimization and sensitivity analysis later on. Hence the portfolio initiation is performed as of 2014-03-02. The length of portfolio holding period than will constitute 1882 days. At every rebalancing date 10 coins with the number of historical observations higher or equal to 30 days are chosen through Filtering Method III. The length of 30 days is taken as initial value for look-back (LB) period for optimization basing on the high volatility and low level of predictability of digital coins. Longer periods are assumed to have lower predictability since the huge volatility of cryptocurrencies. Shorter periods though could be not enough to estimate the proper weights for the optimization, because usually fewer number of observations can overestimate the model. Since the assets utilized in the study deviate from the normal definitions, the assumptions made regarding look-back (LB) period should be verified, hence this parameter will be checked for robustness in the sensitivity analysis part. Its value will be changed to 15 days and 60 days. In case of look back (LB) equal to 15, 30, 60 days, the start date of the portfolio optimization will be 2014-03-02. Figure 6 presents the described look-back (LB) parameter variation for each of the option throughout the time range employed in this study.

Figure 6. Look-back (LB) parameter analysis for Markowitz optimization.



Source: own calculations based on period 2014/03/02 – 2019/04/26. nObs is the number of observations in days, LB is a look-back period for optimization.

The next parameter required for the optimization is the rebalancing window (RB). Taking into consideration all cryptocurrencies characteristics discussed previously in this section, it is decided to conduct rebalancing each 14 days. The motivation behind quite short period of rebalancing (RB) is the fact that this study aims at showing the variety of investment opportunities on the very volatile market, which experience fast and dynamic changes. Hence, the assets and their weights in the portfolio are also going to change dynamically, each 2 weeks. In order to observe the impact of this parameter on the results of the portfolio performance, the sensitivity analysis will also be conducted and the values to check are 7 days and 21 days.

Portfolio optimization is performed in R with the help of package "PortfolioAnalytics". In order to start the optimization, some more parameters should be defined and assigned in the framework of the employed technique. One of the crucial concerns in the optimization is defining the objective. The initial objective in this study is maximization of Information Ratio (IR) in Markowitz framework. Information Ratio is the measure of the risk-adjusted portfolio return compared to a benchmark or risk-free investment.[3] In this study the Information ratio is calculated by dividing the annualized portfolio return by annualized standard deviation of the return assuming the risk-free rate equal to zero and is denoted as IR. The research aims at

[3] This seems to be the most frequently used definitions of IR, see for example: https://www.investopedia.com/ask/answers/010815/what-difference-between-sharpe-ratio-and-information-ratio.asp

finding a profitable strategy, with the IR as high as possible. However, since cryptocurrency market is very volatile, which has been already shown in this paper, an investor may think of minimizing the standard deviation of the digital portfolio. In that case the return could be lower but investment in crypto market is than would be treated safer. Hence, another objective stated is minimizing volatility.

While constructing the rebalanced portfolio within Markowitz framework it is required to set not only the objectives but also constraints, targets, and restrictions on weights. The restrictions for optimization are set as following: full investment is assumed, which means that sum of weights equal to 1, due to the technical need this parameter is set in the range between 0.99 and 1.01; long only restriction, meaning that no negative weights are allowed; minimum weight is 0.01 and maximum weight is 0.6, which suggests that each cryptocurrency out of 10 should be assigned a weight, but as well, this constraint ensures not to allocate too much of the total wealth into one asset, because in the case of cryptocurrencies, Bitcoin may take the prevailing weight due to its high returns in some periods.

Another important parameter for the portfolio optimization would be transaction cost. While undertaking investment into cryptocurrencies, investor should be prepared for incurring fees for the transactions. Since the rebalancing is made quite often, transaction cost is assumed to have a substantial impact on the strategy profitability, especially while increased. Hence, the initial value of the transaction cost is set up to be 1% and then it is going to be increased to 2% in the sensitivity analysis. Even at the prior stage, there appears an obvious assumption that double increase in transaction cost will decrease the return on investment, having all other parameters unchanged, especially, bearing in mind the frequency of portfolio rebalancing. Decreasing the rebalancing window in the sensitivity analysis to 7 days should obviously increase costs and decline the return on such strategy.

Since, there are many parameters for filtering and optimizing taken into account and subject to sensitivity testing, we summarize the combination of all parameters for Markowitz strategies as well as define the default values of those parameters for initial strategy testing. The number of strategies obtained in the process of parameters combination for Markowitz optimization is 2*3*3*2 = 36:

- Optimization objective: maximum Information Ratio (IR), minimum variance (MV) (2 values)

- Lookback period (LB) based on which returns and variance-covariance matrix are estimated: 30 days, 15 days, 60 days (3 values)
- Rebalancing period, period after which new weights for cryptocurrencies are calculated (RB): 14 days, 7 days, 21 days (3 values)
- Transaction costs (TC): 1**%** and 2% (2 values)

The naming convention employed in this research takes into consideration 4 parameters: LB, RB, TC, IR/MV: MarkCap10_LB_RB_TC_IR / MarkCap10_LB_RB_TC_MV.

The primary combinations of initially assumed parameters for Markowitz optimization then would be the following:

- MarkCap10_LB30_RB14_TC1_IR[4]
- MarkCap10_LB30_RB14_TC1_MV[5]
- MarkCap10_LB30_RB14_TC2_IR[6]
- MarkCap10_LB30_RB14_TC2_MV[7]

The portfolio is initiated with the help of function `portfolio.spec()` and optimized with the method `DEoptim` (Mullen et al. 2011). The function `optimize.portfolio()` available in the utilized package optimize the assets' allocation and returns the weights assigned to chosen cryptocurrencies for the particular rebalancing date. The weights obtained in the process stored in the list with the relative crypto names and under relative date.

Having multiplied the returns of the cryptocurrencies participating in the portfolio by the lagged values of their weights, portfolio returns are obtained. Since the weights are obtained only for the rebalancing dates and the aim of the research is to monitor the portfolio behavior on the daily basis, the weights of the assets between the rebalancing dates $w_{i,t}$ are adjusted in relation with their returns for that dates:

[4] Portfolio with look-back (LB) 30 days, rebalancing window (RB) 14 days, transaction cost (TC) 1% and optimization objective of maximizing Information Ratio (IR).
[5] Portfolio with look-back (LB) 30 days, rebalancing window (RB) 14 days, transaction cost (TC) 1% and optimization objective of minimizing variance (MV).
[6] Portfolio with look-back (LB) 30 days, rebalancing window (RB) 14 days, transaction cost (TC) 2% and optimization objective of maximizing Information Ratio (IR).
[7] Portfolio with look-back (LB) 30 days, rebalancing window (RB) 14 days, transaction cost (TC) 2% and optimization objective of minimizing variance (MV).

$$R_{0,T}^{(p)} = \prod_{t=1}^{T}\left(1 + \sum_{i=1}^{N} w_i r_{i,t} - \Delta W_t^R * TC\right) - 1$$

$$\Delta W_t^R = \frac{\sum_{i=1}^{N}\left|x_{i,t}\right|}{PV_t}$$

$$x_{i,t} = w_{i,t} - w_{i,t-1}$$

$$w_{i,t-1} = w_{i,t-2} * (1 + r_{i,t-1})$$

$$w_{i,t} = \frac{\left(1 + r_{i,t}\right) * w_{i,t-1}}{1 + \sum w_{i,t-1} * r_{i,t}} - Markowitz\ rebalancing$$

$$w_{i,t} = \frac{1}{N} = \frac{1}{10} - equally\ weightes\ rebalancing$$

$$w_{i,t} = \frac{MC_{i,t}}{\sum_i^N MC_{i,t}} - market\ cap.weighted\ rebalancing$$

where:

N – number of assets, $i \in \{1, \dots, N\}$,

T – total number of intervals (duration of the investment) in the period between 0 and T, $t \in \{1, \dots, T\}$,

$R_{0,T}^{(p)}$ – simple rate of return of the portfolio in the period between 0 to T, $R_{0,T} = \frac{P_T - P_0}{P_0}$

$w_{i,t}$ – percent weight of i-th asset in the whole portfolio in the period of t,

$r_{i,t}$ – rate of return of i-th asset in the period t,

ΔW_t^R – portfolio allocation change (range from 0% up to 100%) between day t -1 and day t, where day t is the reallocation day.

TC – transactional costs at the level of 1% or 2%,

$x_{i,t}$ – allocation change (range from 0% up to 100%) for i-th crypto between day t -1 and day t, where day t is the reallocation day,

$MC_{i,t}$ – market cap of the i-th asset on day t,

MC_t – sum of all market caps on day t.

In order to observe the behavior of the portfolio, equity lines for the Markowitz optimized strategy together with equity lines for the benchmarks are produced according to the formula:

$eq.line_t = (1 + r_1)(1 + r_2) \dots (1 + r_t)$, where r_t is a simple daily return.

Moreover, apart from looking at equity lines on the plot and the resulting profit or loss, it is essential to estimate certain performance measures, in order to be able to make sensible conclusions about strategies' performance. Hence, the measures used for comparison in this research are the following:

- annualized rate of return of the strategy (aRC)

$$aRC = \left(\left(1 + \frac{P_{i,T}}{P_{i,0}} \right)^{\frac{365}{T}} - 1 \right) * 100,$$

where $P_{i,T}$ is a price of i-th asset at the end of interval T

- annualized standard deviation of the strategy daily returns (aSD)

$$aSD = \sqrt{\frac{365}{T} \sum_{t=1}^{T} (r_{i,t} - \bar{r})^2} *100$$

- Information Ratio (IR), which describes the relation of the portfolio annualized rate of return to the annualized volatility of the return (IR)

$$IR = \frac{aRC}{aSD}$$

- Maximum Drawdown (MD), which is the maximum loss from a peak to a minimum of a portfolio before a new peak is attained (MD)[8]

$$MD(T) = max_{\tau \in [0,T]} (max_{t \in [0,\tau]} R_{i,T}^{(p)} - R_{i,\tau}^{(p)}) *100$$

- the relation of Information Ratio to maximum drawdown (IRMD)

$$IRMD = \frac{IR}{MD}$$

- the relation of product of IR and annualized return to the maximum drawdown (IRaRCMD):

$$IRaRCMD = IR * \frac{aRC}{MD}$$

[8] https://www.investopedia.com/terms/m/maximum-drawdown-mdd.asp

3.3.2 Benchmarks definition

Since the cryptocurrencies market is very innovative and unstable, and there is not still much proof if it may function in accordance with some traditional market standards, the performance of Markowitz optimized strategies would be quite difficult to assess in the context of attractiveness to investors. In order to be able make conclusions regarding the results of the portfolio optimization, it is necessary to compare its behavior with the benchmarks as it is usually done within classical investment.

Although, in this paper he cryptocurrencies market is considered as a separate asset class and investment made solely within it, quite interesting benchmark for the general comparison of digital and classical markets would be index S&P500. This is one of the most known US indices, which incorporates 500 entities with the highest market capitalization. The index is used as a benchmark for many researches concerning investment opportunities on the classical markets. Hence, buy-and-hold investment strategy is initiated for that index, which represents traditional wide passive investment on the regulated market and called "SPX" in this study. The starting date of the data downloaded is 2014-01-01 and it ends up on 2019-04-26[9]. The returns are calculated, and the equity line is produced for the buy-and-hold strategy starting as of 2014-03-02 in order to have the equity line of the same length as the equity line for Markowitz optimization.

The following benchmark considered is the buy-and-hold investment in Bitcoin. The interesting point by this strategy is the fact that Bitcoin has always occupied the biggest part of the cryptocurrency market in terms of market capitalization, which suggests that it describes the movement of the market in a way. Figure 5 provides the proof as well as justifies the nature of taking this investment under consideration. Mostly, Bitcoin is considered for investment in cryptocurrencies or added to the diversified portfolio. Hence, it would be interesting to observe if the single Bitcoin investment could outperform the Markowitz optimized portfolio consisted of 10 coins selected according to highest market capitalization including Bitcoin itself. This investment in the research is indicated as "BTC_Single".

Further considering market capitalization as a significant factor in investment decisions, it would be interesting to compare Markowitz results with the strategy of passive investment on digital market is indicated as "MarkCap". In this strategy, weights are assigned in the

[9] Source: https://stooq.pl

proportion of the coins' market capitalization to the sum of market capitalizations of all ten cryptos in the portfolio for each date of rebalancing. For this type of investment transaction cost is also taken into consideration, both initial value of 1% and the second option of 2%.

The last benchmark, which is supposed to be compared to the obtained results of Markowitz optimization is equally weighted investment. We wonder, how the results would differ if we don't use neither Markowitz framework nor market capitalization principle. In this benchmark we assume that ten coins selected through the filtering criteria deserve the same weights, namely 10 % of the total wealth is taken by every crypto. This strategy is called "EqualWeights" in this study and aims at revealing the difference between applying Markowitz optimization, where weights are assigned on the basis of sophisticated calculations and the simple passive strategy of assuming equal weights. In the case of classical assets, such strategy may result in the less profitable performance, because when utilizing naive technique of passive investment, namely, ignoring past returns, market share and standard deviation of assets, the concept of rational assets allocation is disturbed. However, it may not be the case within hardly examined digital assets. Transaction cost of 1% and 2% are also applied for this benchmark.

To sum up, there are 4 benchmarks in this research, suggested basing on the willingness to assess the cryptocurrencies market investment opportunities from the point of view of traditional techniques:

1. S&P500 index buy-and-hold (SPX)
2. Bitcoin buy-and-hold (BTC_Single)
3. Market capitalization weighted investment (MarkCap)
4. Equally weighted investment, where each coin's weight is 10% (EqualWeights)

All parameters combinations constitute 36 variations of the strategies. Four primary scenarios and 16 scenarios for the sensitivity analysis will be reflected on the plot with 5 equity lines: Markowitz and 4 benchmarks (SPX, BTC_Single, MarkCap and EqualWeights). The plots will be shown in Section 3 (Figure7- Figure10). Strategies performance measures are also produced for every group of portfolios. The results and foundations are presented in the next section.

4. Results

4.1 Results of primary strategies

Once all the required preparation was done, the results for the strategies were generated. First of all, four initial scenarios developed and produced with the assumed combination of parameters are to be considered and assessed: LB30_RB14_TC1_IR, LB30_RB14_TC1_MV, LB30_RB14_TC2_IR, LB30_RB14_TC2_MV. Table 1 presents the statistics on these investments. Each of them is visualized later on the graphs with 3 panels, which show:

- 5 equity lines on the linear scale in order to see the profitability and overall time series behavior,
- 5 equity lines drawn on the logarithmic scale in order to have a closer look at the performance of equity lines through the whole-time range, especially in the periods, which are not visible on the first panel in the beginning of the time range,
- 5 series, showing the drawdowns of portfolio returns for Markowitz and its four benchmarks.

For each of four scenarios the strategies in the analysis are grouped by three in Table 1: Markowitz strategy, EqualWeights and MarkCap, because they are affected by changes in the parameters. BTC_Single and SPX are the benchmarks representing passive investment buy-and-hold, hence they remain entirely constant throughout the study.

The first observation made is the fact that the performance measures for the cryptocurrency's portfolios are extremely high, especially compared to SPX index identifying passive investment on regulated market. The analysis of the results in the Table 1 start with comparison of Markowitz strategies and the suggested benchmarks. The first scenario with the set of parameters LB30_RB14_TC1_IR draws an attention, first of all, to the annualized rate of return, which is positive and equal to 118%, especially in comparison with all benchmarks: EqualWeights – 50%, MarkCap – 52%, BTC_Single – 54% and SPX – 9%. However, for sure, the spectacular aRC is not the key indicator of strategy's performance. What really matters in the optimization evaluation is the Information ratio. Markowitz strategy shows IR equal to 1.44, which is an absolute leader within the considered investment strategies across all four scenarios. The worst Information ratio is incurred by EqualWeights strategy, which also showed the poorest aRC, highest aSD and MD among crypto-based benchmarks, which implies the apparent conclusion about naïve investment not working well on the digital market. MarkCap and BTC_Single have quite similar statistics, especially in terms of IR, however, which seems interesting, is the fact that the IR for Bitcoin's buy-and-hold is still slightly higher than for the portfolio weighted by market capitalization: 0.74 and 0.72 respectively. If examining all other statistics within these two investments, the observation made implies that Bitcoin still performs better by all measures.

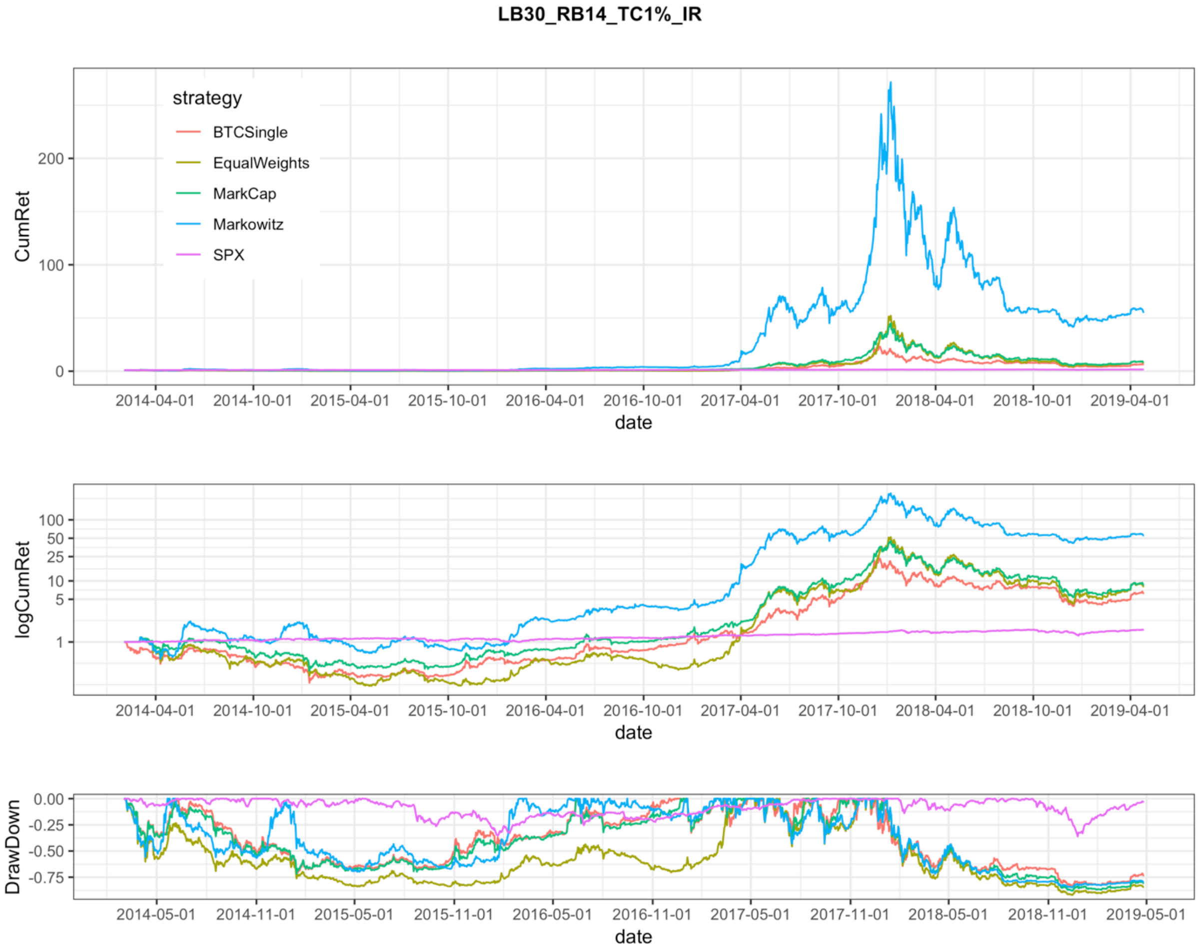
Table 1. Statistical measures for four primary scenarios of investment strategies.

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB14_TC1_IR							
Markowitz	118.3	81.9	84.7	1.44	1.70	2.02	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB30d_RB14_TC1_MV							
Markowitz	69.0	77.1	82.1	0.90	1.09	0.75	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB30d_RB14_TC2_IR							
Markowitz	64.3	83.5	89.5	0.77	0.86	0.55	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB30d_RB14_TC2_MV							
Markowitz	54.1	81.3	89.4	0.67	0.75	0.40	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

1. Source: own calculations based on period 2014/03/02 – 2019/04/26.
2. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD -the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days.
3. Markowitz – optimized portfolio within parameters noted in the caption of each subgroup consisted of 3 rows, EqualWeights – strategy, where optimized weights are replaced by equal weights, 0.1 for each of 10 chosen coins, MarkCap – strategy, where weights are allocated in accordance with the relation of coin's market capitalization to the sum of ten coins' market capitalization.
4. aRC, aDS, MD reflect the percentage change.

Another interesting observation is that the IR for SPX (0.72) is close to the values of IR for MarkCap and BTC_Single. This is quite surprising but can be explained by the relation of aRC and aSD of these investments. Both of these measures are relatively low for SPX, which opposite to huge values of annualized standard deviation and annualized return of crypto portfolios. But if considering the IRMD measure, SPX outperforms all crypto strategies and showing value of 3.62, whereas the second biggest result belongs to Markowitz (1.70). Such difference can be explained by very low maximum drawdown on index S&P500, supports the idea of this strategy be much safer in terms of probability of loss. Markowitz optimized portfolio is considered the most successful within the first scenario of investment. The winner among the benchmarks is the BTC_Single's performance but still not as good as the portfolio optimized within certain assumptions undertaken dynamically on the crypto market. The visualization of results of the first investment scenario is presented on Figure 7.

Figure 7. Visualization of results for scenario LB30_RB14_TC1_IR



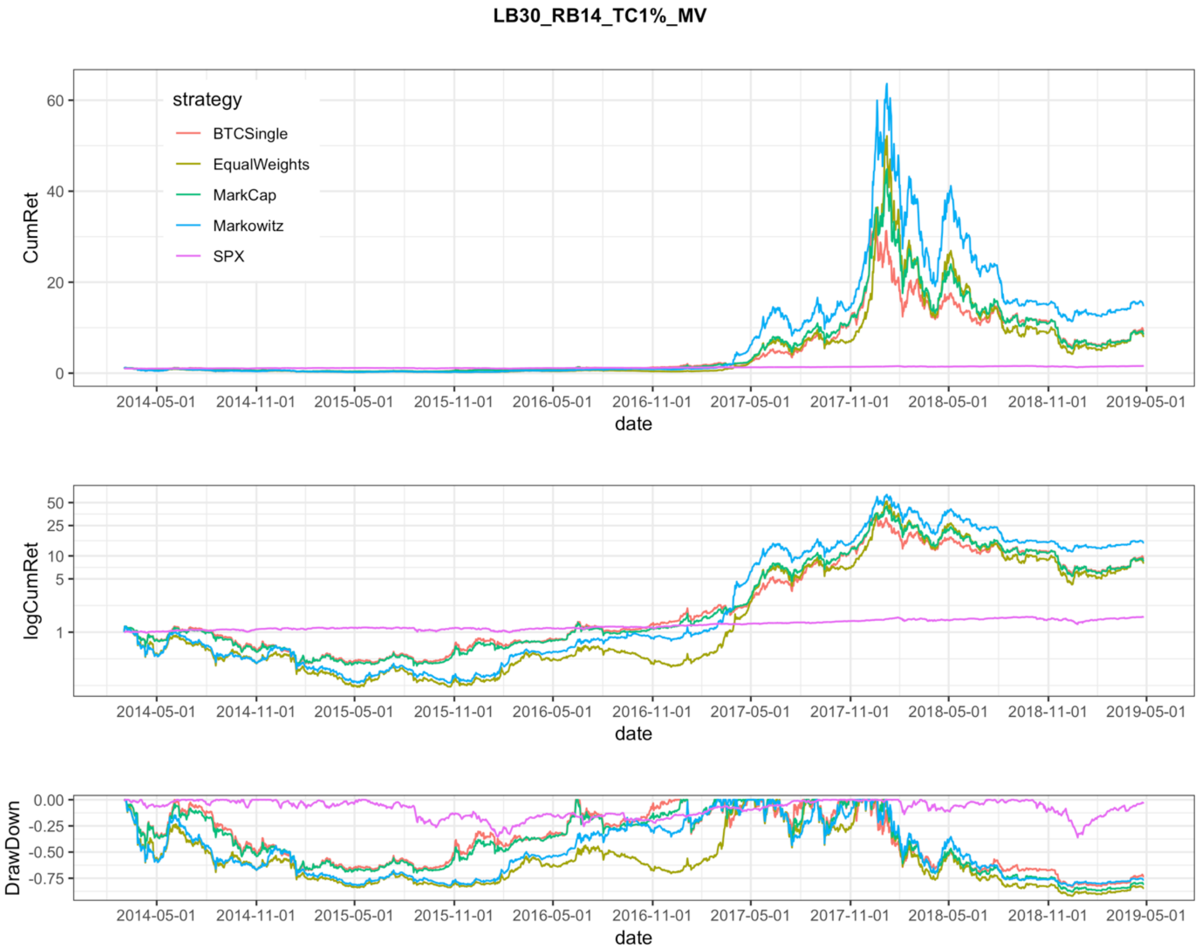
Source: own calculations based on period 2014/03/02 – 2019/04/26. 1 Panel shows the equity lines of 5 strategies (Markowitz + 4 benchmarks) for scenario with parameters LB30_RB14_TC1_IR, 2 Panel presents the changes in these 5 equity lines on the logarithmic scale, 3 Panel shows the series of drawdowns for every strategy

Having looked at the panel one of the Figure 7, it is obvious from the first sight, that the return on the strategy optimized within mean-variance framework is increased by more than 50 times since the first date of rebalancing. All the other benchmarks results look modest in comparison with Markowitz performance. Due to a huge rise in the period started 2017-04-01, the first part of equity lines is invisible on the first panel. Hence, the logarithmic scale is involved. All 5 strategies are drawn and show the relative changes of the cumulated returns through the whole-time range. The strong correlation between BTC_Single and MarkCap is observable on first and second panels, supporting the observation noted before. The log scale also shows the difference in the nature of two kinds of investments: traditional index and digital assets. SPX equity line as well as its drawdown line presented on the third panel look much less volatile than the portfolios consisted of cryptos. This is though supported by the values in Table 1, regarding the maximum drawdown and annualized standard deviation. If considering the EqualWeights strategy, then we can see that second and third panel show that this investment behaves poorly in comparison with all other benchmarks. It crossed the equity line of SPX from the below the last out of all other strategies, it possesses higher drawdown, especially in years 2015-2016. The graphical presentation of the first investment scenario supports the statistical table in all aspects.

Now, for the same LB, RB and TC parameters, objective of minimizing volatility is reviewed: LB30_RB14_TC1_MV. Since the values of parameters for optimization stay unchanged in the second scenario apart from the objective denoted MV, all benchmarks' statistics stay the same, whereas the results of the Markowitz strategy decrease significantly. Since the weights were allocated in order to ensure the lowest possible volatility, the aSD is decreased while comparing to the result for objective of maximization IR: 77% versus 82%, which is, actually, not a big change, because the value of this measure is still very high in relation to the standard deviation of traditional benchmark SPX. If then to have a look at the annualized return of Markowitz portfolio, we can see that it is almost twice as lower than in the first scenario. The changes in the values of aSD and aRC after switching objective are inconsistent.

The maximum drawdown does not change significantly, namely, slight decrease from 85% to 82%, the IR drops from 1.44 to 0.89 sharply and IRMD from1.70 to 1.09. Nothing changes in the relation between four benchmarks' statistics ranking, Bitcoin buy-and-hold still outperforms the rest three. However, there is a change in the relation between Markowitz portfolio performance and its benchmarks, because in this scenario, opposite to the previous, where the choice of the strategy was obvious, the question about the most attractive strategy becomes more disputable. The aSD is lower for BTC_Single than for LB30_RB14_TC1_MV. There is a slight superiority of the value of IR for the Markowitz portfolio over Bitcoin's portfolio, which can be treated negligible in comparison to the lower aSD experienced by BTC_Songle portfolio. Figure 8 presents the visualization of second scenario investment.

Figure 8. Visualization of results for scenario LB30_RB14_TC1_MV



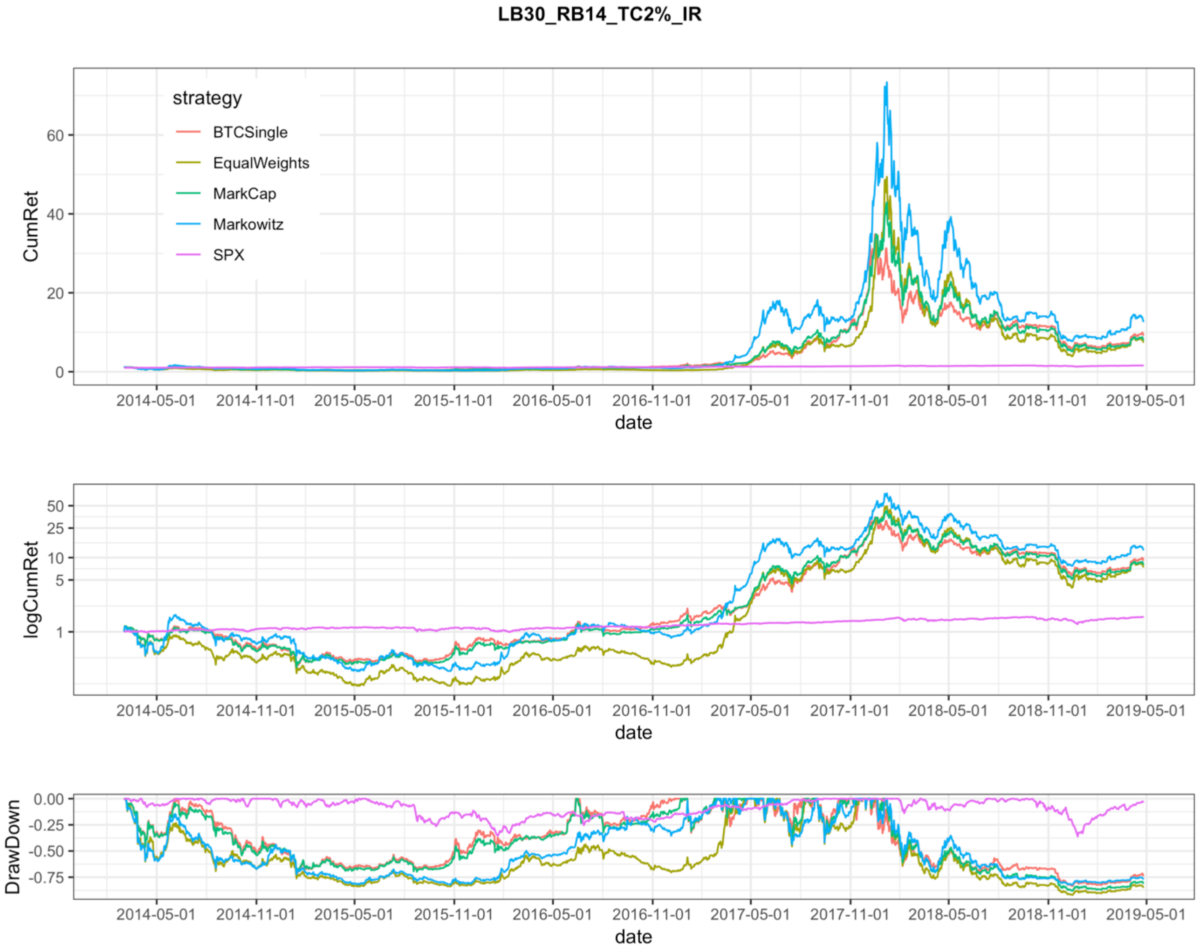
Source: own calculations based on period 2014/03/02 – 2019/04/26. 1 Panel shows the equity lines of 5 strategies (Markowitz + 4 benchmarks) for scenario with parameters LB30_RB14_TC1_MV, 2 Panel presents the changes in these 5 equity lines on the logarithmic scale, 3 Panel shows the series of drawdowns for every strategy.

The plotting of results of the second investment case is quite similar to the previous one. The observable difference is the performance of Markowitz portfolio. The return on the digital portfolio presents around sixteen-time increase, which is much less than in the first one. The log scale changes in equity line behavior during the time horizon is also noted. It crosses the value 1 and the flat equity line of SPX from the below, meaning that it started to earn, in the end of year 2016, which is much later than it did in the first scenario, where it happened in the beginning of year 2016. The values of the drawdown values for the optimized portfolio are higher and converge to the ones of EqualWeights portfolio. The general performance of the Markowitz strategy has got worth, when switching between IR and MV optimization objectives.

For scenario 3 and 4 we still optimize on the look-back 30, rebalancing window 14, IR and MV objectives respectively but now transactional costs are doubled from 1% to 2%: LB30_RB14_TC2_IR, LB30_RB14_TC2_MV. The most severe impact is experienced by Markowitz strategy with optimization objective IR: almost double decrease in aRC from the value of 118% to 64%, increase in aSD, increase in MD and almost double decrease of IR from 1.44 to 0.77 suggest that transaction costs has a strong impact on the strategy's profitability. While taking about benchmarks, where the costs increase was also implemented, there is no extreme changes. All the measures worsened a little bit but the relation between their general attractiveness did not change too much. Bitcoin still has highest aRC, lowest MD and highest IR among for types of passive investments. However, when referring specifically to scenario three (IR objective), even after the Markowitz annualized rate of return and Information ratio sharp declines, it is still the best one in terms of most statistics. The IR of 0.77 is higher than SPX 0.71 or BTC_Single 0.74 values, however not much which makes the equity lines look very similar for that scenario. Visualization is presented on Figure 9. Despite the serious drop in the final return of the Markowitz portfolio, the visualized equity lines support the idea of optimized portfolio still being quite attractive especially within the investment scenario three, because other strategies perform worse. Another curious observation on the Figure 9 is that in the second part of time horizon the changes in returns shown on the log scale panel as well as the drawdowns described in the third panel are very correlated. Moreover, their values also are very similar including Markowitz, which follows pretty the same pattern as the benchmarks for the comparison.

The impact of TC rise is also harmful for the Markowitz framework with MV optimization objective. Scenario four suggests that while having a transaction cost equal to 2%, the results of its strategy go below the measures of BTC_Single in terms of all measures. The Information ratio is lower than for Bitcoin, 0.67 versus 0.74. Moreover, SPX also outperforms LB30_RB14_TC2_MV in terms of higher IR (0.72), much lower aSD (13%) and much lower MD (20%). At this situation, an investor either chooses buy-and-hold on the regular market with lower risk and lower return or buy-and-hold on the innovative market with high risk and relatively high return having almost the same IR level.

Figure 9. Visualization of results for scenario LB30_RB14_TC2_IR



Source: own calculations based on period 2014/03/02 – 2019/04/26. 1 Panel shows the equity lines of 5 strategies (Markowitz + 4 benchmarks) for scenario with parameters LB30_RB14_TC2_IR, 2 Panel presents the changes in these 5 equity lines on the logarithmic scale, 3 Panel shows the series of drawdowns for every strategy.

The level of aSD (81%) though is almost equal to the standard deviation of Markowitz strategy (82%) in the first scenario where optimizing IR within TC 1%, and also is higher than for optimizing MV with TC 1%, which implies no significant improvement of riskiness of the strategies while switching optimization objectives and increasing transaction costs. The results are presented on the Figure 10 below.

On the Figure 10, we can see that all digital portfolio equity lines behave very similar, especially starting with second quarter 2017 in terms of all three panels on the plot. Such foundation suggests that increasing transaction costs can wipe out the excess return of optimized portfolio within the Markowitz framework and also equalize the results of strategies produced by the dynamic asset allocation and passive investment.

Figure 10. Visualization of results for scenario LB30_RB14_TC2_MV

Source: own calculations based on period 2014/03/02 – 2019/04/26. 1 Panel shows the equity lines of 5 strategies (Markowitz + 4 benchmarks) for scenario with parameters LB30_RB14_TC2_MV, 2 Panel presents the changes in these 5 equity lines on the logarithmic scale, 3 Panel shows the series of drawdowns for every strategy.

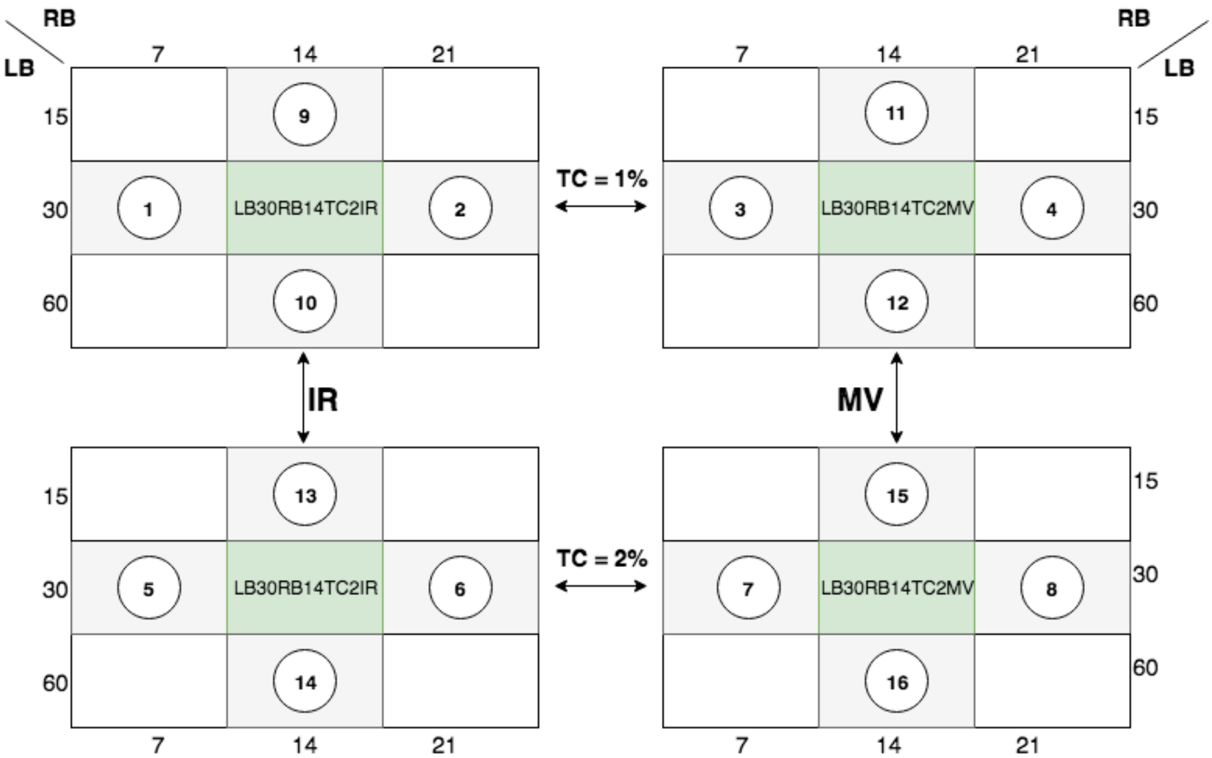
Generally, results for all 4 scenarios suggest that portfolio optimized within Markowitz framework outperforms the passive investment benchmarks in most cases, even under increased transaction costs for IR objective. The objective of IR maximization turns out to bring more profit than minimizing volatility, which was predictable. Passive cryptocurrencies benchmarks are strongly correlated, especially BTC_Single and MarkCap due to the share of Bitcoin in the total market capitalization. The lowest return among crypto benchmark is incurred by EqualWeights along all 4 scenarios. The worst drawdown for all cryptocurrencies strategies is experienced in the period between third quarter 2014 and end of first quarter 2017. Performance of SPX benchmark, as expected, is much more stable in comparison with digital portfolios. Its equity line looks quite flat but also slightly increasing. The drawdown line is also less volatile and generally less severe than lines for cryptocurrencies investments. However, SPX shows the

IR, very close to the values produced by a number of crypto benchmarks, especially taking into consideration higher costs.

4.2 Sensitivity analysis

In this part the sensitivity analysis on parameters for optimization will be conducted. Strategies performance will be checked for robustness depending on the parameters change. There are 4 parameters: look-back period, rebalancing window, transaction costs and the optimization objective. The dependencies in switching between objectives and transaction costs increase were already addressed in the results part, where LB and RB remained stable. However, the change in parameters LB and RB will be done for both variations of TC and optimization objectives. In order to simplify the navigation between switching parameters diagram is created and presented on Figure 11.

Figure 11. Combination of parameters for sensitivity analysis



Source: own calculations based on period 2014/03/02 – 2019/04/26. RB – rebalancing window of strategies portfolio optimization, LB – look-back period for optimization, IR – optimization objective of maximizing Information Ratio, MV - optimization objective of minimizing variance.

For the sake of better understanding of the impact of the changes in the parameters on strategies performance, it is decided not to review 32 strategies, but stick to 16, which are denoted by numbers from 1 to 16 on the Figure 11. Four groups of four combinations of parameters will be reviewed in this section.

First sensitivity scenario: strategies 1:4

First, we change the rebalancing window and examine its impact on strategies performance for TC equal to 1%. Hence, the strategies denoted as: LB30_RB7_TC1_IR; LB30_RB21_TC1_MV; LB30_RB7_TC1_IR; LB30_RB21_TC1_MV. For simplicity, looking at Figure 11, we take into consideration strategies which result in the combination of parameters denoted by 1 to 4 as a first scenario for analysis. Table 2 summarizes the statistics on the results of the first scenario in the sensitivity analysis.

Having looked at Table 2, it is easy to claim that the winner in terms of highest annualized rate of return (216%) and highest Information ratio (2.28) is the Markowitz strategy with the rebalancing window increased from 14 to 21 days. However, the aSD is the highest among all four groups of strategies in the scenario one of sensitivity analysis, which implies this type of investment return be corresponding to such abnormally high standard deviation (95%). What is interesting here is that in comparison with the primary strategy LB30_RB14_TC1_IR , where the look-back was 14 days, the aRC almost doubled (from 118% to 216%). A very good increase in aRC (163%) and IR (1.73) is shown by the strategy LB30_RB21_TC1_MV (4 on Figure 11), where the look-back period is also changed from 14 to 21. The result of this strategy is also significantly outperforming the primary results where aRC was 69% and IR 0.9. The results of the Markowitz strategies for rebalancing window (RB) changed for 7 days for both objectives IR and MV (1 and 3 on Figure 11) have very similar results to strategies with the look-back (LB) 14 days, which were considered as primary ones.

The results of benchmarks also improved, both with LB 7 and LB 21. None of them still could outperform the Markowitz within their groups (Markowitz + 4 benchmarks). Although, there is one benchmark which draws the attention in Table 2. This is the EqualWeights strategy for the rebalancing period 21 days has quite high aRC 87% and IR 0.95. The worst performing strategies though is the MarkCap benchmark for 21 days RB and EqualWeights for 7 days RB. Their results did not get worse in comparison with the statistical values of primary strategies with RB 14 days but look not as much attractive as Markowitz strategies and some of benchmarks. In the sensitivity analysis the BTC_Single as well as SPX continue remaining stable, and in this scenario their results are very far from the Markowitz performance with LB 21 days.

Table 2. Statistical measures for *first* sensitivity analysis scenario of investment strategies.

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB7_TC1_IR							
Markowitz	116.0	80.6	82.2	1.44	1.75	2.03	1882
EqualWeights	59.0	87.5	92.0	0.67	0.73	0.43	1882
MarkCap	50.6	72.6	88.2	0.70	0.79	0.40	1882
MarCap10_LB30d_RB21_TC1_IR							
Markowitz	215.9	94.8	80.3	2.28	2.83	6.12	1882
EqualWeights	86.7	91.0	92.2	0.95	1.03	0.90	1882
MarkCap	53.5	72.6	88.1	0.74	0.84	0.45	1882
MarCap10_LB30d_RB7_TC1_MV							
Markowitz	66.3	75.2	84.6	0.88	1.04	0.69	1882
EqualWeights	59.0	87.5	92.0	0.67	0.73	0.43	1882
MarkCap	50.6	72.6	88.2	0.70	0.79	0.40	1882
MarCap10_LB30d_RB21_TC1_MV							
Markowitz	163.3	94.5	74.6	1.73	2.32	3.78	1882
EqualWeights	86.7	91.0	92.2	0.95	1.03	0.90	1882
MarkCap	53.5	72.6	88.1	0.74	0.84	0.45	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

1. Source: own calculations based on period 2014/03/02 – 2019/04/26.
2. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD - the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days.
3. Markowitz – optimized portfolio within parameters noted in the caption of each subgroup consisted of 3 rows, EqualWeights – strategy, where optimized weights are replaced by equal weights, 0.1 for each of 10 chosen coins, MarkCap – strategy, where weights are allocated in accordance with the relation of coin's market capitalization to the sum of ten coins' market capitalization.
4. aRC, aDS, MD reflect the percentage change.

Second sensitivity scenario: strategies 5:8

Second scenario suggests again to change the LB period but now together with the increased transaction costs equal to 2%. The results of primary strategies showed the declining when TC was enlarged. Table 3 reveals the statistical measures of the second scenario in sensitivity analysis where the LB and TC parameters are to be changed.

Table 3. Statistical measures for *second* sensitivity analysis scenario of investment strategies.

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB30d_RB7_TC2_IR							
Markowitz	60.3	83.6	90.5	0.72	0.80	0.48	1882
EqualWeights	54.6	87.5	92.2	0.62	0.68	0.37	1882
MarkCap	47.4	72.6	88.5	0.65	0.74	0.35	1882
MarCap10_LB30d_RB21_TC2_IR							
Markowitz	88.8	87.7	91.3	1.01	1.11	0.98	1882
EqualWeights	85.1	90.9	92.3	0.94	1.01	0.86	1882
MarkCap	52.5	72.6	88.2	0.72	0.82	0.43	1882
MarCap10_LB30d_RB7_TC2_MV							
Markowitz	50.0	81.5	88.3	0.61	0.69	0.35	1882
EqualWeights	54.6	87.5	92.2	0.62	0.68	0.37	1882
MarkCap	47.4	72.6	88.5	0.65	0.74	0.35	1882
MarCap10_LB30d_RB21_TC2_MV							
Markowitz	100.8	92.5	90.0	1.09	1.21	1.22	1882
EqualWeights	85.1	90.9	92.3	0.94	1.01	0.86	1882
MarkCap	52.5	72.6	88.2	0.72	0.82	0.43	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

1. Source: own calculations based on period 2014/03/02 – 2019/04/26.
2. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD - the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days.
3. Markowitz – optimized portfolio within parameters noted in the caption of each subgroup consisted of 3 rows, EqualWeights – strategy, where optimized weights are replaced by equal weights, 0.1 for each of 10 chosen coins, MarkCap – strategy, where weights are allocated in accordance with the relation of coin's market capitalization to the sum of ten coins' market capitalization.
4. aRC, aDS, MD reflect the percentage change.

Again, the Markowitz strategies with RB 21 days outperform those with RB 7 days, however, in this case the one with objective MV shows better results, aRC equal to 101%, IR equal to 1.09 and highest IRMD equal to 1.22, than the one with the objective IR, aRC 89% and IR – 1.01. There appears an unexpected observation about the fact that for the strategy with RB 7 days, TC 2% and objective MV (7 on Figure 11) shows lower aRC 50% than its benchmarks: 55% for EqualWeights and 47% for MarkCap. The IR for Markowitz strategy is 0.69 compared to 0.68 and 0.74 for the benchmarks respectively. In this case, the buy-and-hold passive investment benchmarks SPX and BTC_Single outperform in terms of IR, 0.72 and 0.74 respectively.

For IR objective the results of LB equal to 21 days are better than for 14 days and 7 is worse than 14 days. Exactly the same dependency is noticed for the MV objective. The maximum drawdowns values are huge for all Markowitz strategies.

Third sensitivity scenario: strategies 9:12

Right now, the look-back period for portfolio optimization is addressed. The primary strategies were given the value of this parameter equal to 30 days. In this scenario, the LB parameter should be changed to 15 days and 60 days, while having TC equal to 1%, shown in Table 4.

Table 4. Statistical measures for *third* sensitivity analysis scenario of investment strategies.

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB15d_RB14_TC1_IR							
Markowitz	94.3	81.6	81.9	1.16	1.41	1.33	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB60d_RB14_TC1_IR							
Markowitz	98.1	77.9	83.2	1.26	1.51	1.49	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB15d_RB14_TC1_MV							
Markowitz	34.9	76.3	92.2	0.46	0.50	0.17	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
MarCap10_LB60d_RB14_TC1_MV							
Markowitz	94.0	76.5	80.2	1.23	1.53	1.44	1882
EqualWeights	50.4	86.9	91.9	0.58	0.63	0.32	1882
MarkCap	52.4	72.6	87.9	0.72	0.82	0.43	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

1. Source: own calculations based on period 2014/03/02 – 2019/04/26.
2. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD - the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days.
3. Markowitz – optimized portfolio within parameters noted in the caption of each subgroup consisted of 3 rows, EqualWeights – strategy, where optimized weights are replaced by equal weights, 0.1 for each of 10 chosen coins, MarkCap – strategy, where weights are allocated in accordance with the relation of coin's market capitalization to the sum of ten coins' market capitalization.
4. aRC, aDS, MD reflect the percentage change.

Description of Table 4 is started with the worst resulting strategy, namely the 11th strategy on Figure 11, with LB 15 days, RB 14 days, TC 1 % and the objective of minimizing variance MV. The aRC – 35%, MD – 92% and IR 0.46. Huge maximum drawdown and very small aRC compared to other strategies even majority of benchmarks suggest that the short look-back period does not cope with the huge variance and is not profitable strategy in terms of IR. The best performance is shown by 10th strategy on Figure 11, where the look-back period constitutes 60 days and the objective of optimization is maximizing Information ratio (aRC 98%, aSD 78%, IR 1.26). However, its results are worse than the strategy from primary scenarios for LB equal to 30 (see Table 1).

Fourth sensitivity scenario: strategies 13:16

Look-back period change is still addressed but now together with the TC equal to 2%. The results are presented in Table 5.

Table 5. Statistical measures for *forth* sensitivity analysis scenario of investment strategies.

	aRC	aSD	MD	IR	IRMD	IRaRCMD	nObs
MarCap10_LB15d_RB14_TC2_IR							
Markowitz	58.0	86.5	88.8	0.67	0.76	0.44	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB60d_RB14_TC2_IR							
Markowitz	58.2	80.8	89.4	0.72	0.81	0.47	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB15d_RB14_TC2_MV							
Markowitz	46.6	84.0	90.0	0.55	0.62	0.29	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
MarCap10_LB60d_RB14_TC2_MV							
Markowitz	61.6	79.9	87.3	0.77	0.88	0.54	1882
EqualWeights	48.3	86.9	92.0	0.56	0.60	0.29	1882
MarkCap	50.7	72.6	88.1	0.70	0.79	0.40	1882
BTC SPX							
BTC_Single	54.5	73.5	83.4	0.74	0.89	0.48	1882
SPX	9.5	13.2	19.8	0.72	3.62	0.34	1298

1. Source: own calculations based on period 2014/03/02 – 2019/04/26.

2. aRC – annualized rate of return, aSD – annualized standard deviation, MD – maximum drawdown, IR – Information ratio, IRMD - the relation coefficient of IR to MD, IRaRCMD – the relation coefficient of IR multiplied by aRC to MD, nObs – number of observations in days.
3. Markowitz – optimized portfolio within parameters noted in the caption of each subgroup consisted of 3 rows, EqualWeights – strategy, where optimized weights are replaced by equal weights, 0.1 for each of 10 chosen coins, MarkCap – strategy, where weights are allocated in accordance with the relation of coin's market capitalization to the sum of ten coins' market capitalization.
4. aRC, aDS, MD reflect the percentage change.

Table 5 reveals very important observations about the fact that when manipulating look-back period under the condition of increased transaction costs. Both cases where LB is changed to either 60 or 15 days, having TC equal to 2%, the results of Markowitz strategies performance are very similar to their benchmarks. Even though, there is no spectacular winner in this scenario neither among Markowitz, nor among benchmarks, a remark about general tendency to better performance can be done. The LB parameter changed to be 60 days implies slightly better results of Markowitz portfolios. For strategies 14 and 16 on Figure 11, the aRC measure is 58% and 62%, MD 89% and 87% and IR 0.72 and 0.77 respectively. Basing on these values, it is stated that the Markowitz strategy 16 is considered to be winning, within the following parameters: LB60_RB14_TC2_MV. The benchmarks for groups of strategies with LB equal to 15 turn out to outperform the Markowitz portfolios in terms of lower aSD and higher IR. For instance, LB15_RB14_TC2_MV Markowitz performance was assessed with IR 0.55, MD 90% and aRC 0.46, whereas for the same set of parameters the benchmark EqualWeights values for those statistics are (IR 0.56, MD 92%, aRC 48.3), MarkCap (IR 0.70, MD 88%, aRC 51%), BTC_Single (0.74, MD 83% and aRC 54%). Obviously, such combination of parameters would not attract an investor aiming at excess return.

To sum up the sensitivity analysis of portfolio performance on cryptocurrencies, we point out 4 leading strategies for each of four sensitivity scenarios:

(2) LB30_RB21_TC1_IR

(8) LB30_RB21_TC2_MV

(10) LB60_RB14_TC1_IR

(16) LB60_RB14_TC2_MV

The strategy which outperforms any other in this study is (2) with longer rebalancing period than initially assumed and the strategy aiming at maximizing risk-adjusted return through setting objective of max IR. The results are generally also improved when the look-back period is increased from 30 to 60 days. Transaction costs have negative impact on performance of Markowitz strategies and decrease the excess return of this strategies over benchmarks. All the plots for the strategies discussed in this section are attached in Appendix 1 and under the relative number as they are named on Figure 11.

5. Conclusions

The new digital financial instruments appeared lately on the market brought an attention to their properties and investment opportunities. Many theories are being suggested about cryptocurrencies characteristics as well as their future. However, the fact is that the crypto market is continuously growing and developing together with the technology behind digital coins, namely, blockchain. The market capitalization of the digital assets reaches the value of over 250 billion dollars and there are more and more areas involving crypto transactions. Since the acceptance of cryptocurrencies is so wide and there been issued studies which reveal interesting and innovative characteristics of digital coins, especially, their statistical measures and abnormal price variations, the question about the potential investment opportunity arises.

Hence, the aim of this study was to treat the cryptocurrency market as a new asset class and verify if investment techniques once found and successfully applied on traditional markets could also enhance the profit from cryptocurrencies portfolio. The key was to reflect the classical methodology, hence Markowitz portfolio optimization was chosen for the study. This research emphasizes the importance of examining many different coins and the crypto market as whole, in order to be able to wrap up the conclusions about the crypto world but not only about Bitcoin's extraordinary behavior. When researchers address adding Bitcoin or several other most popular coins to the portfolio of standard assets, they treat them as single alternative instruments but not the instruments belonging to a new and credible market, which possess integrity and common features.

The data for the research was studied in detail, the filtering of coins has been done in several methods. Eventually, filtering was based on choosing ten coins with highest market capitalization for every rebalancing date (Filtering Method III). Portfolio consistent of filtered cryptocurrencies was initiated and then rebalanced. A number of parameters required for Markowitz optimization were assigned under justified assumptions and the benchmarks for strategies performance comparison within each combination of parameters were defined. The aims of the research stated in the beginning of the paper can be answered the following:

1. Generally, throughout the whole study Markowitz strategies ended up with significant profit. It varied depending on the set of optimization parameters, and in most cases none

of four benchmarks could reach the results produced by Markowitz optimization. It implies that the conventional framework of optimization was successfully replicated onto the cryptocurrency market.

2. The primarily assumed parameters produced the strategies with attractive results in terms of annualized rate of return and Information ratio, especially the strategies, which aimed at maximizing Information Ratio. The statistical measures confirmed the abnormal cryptocurrencies behavior through time. The annualized standard deviation and maximum drawdown values for all strategies including Markowitz and crypto benchmarks were huge, especially when compared with S&P500 index measures.
3. Two objectives were employed in order to produce portfolios. The one which maximized the Information ratio was prevailing at having better results for different set of parameters combination, especially in primary scenarios. However, when sensitivity analysis was done, it turned out that the strategy success also depends on various combination of parameters but not only on the optimization objective. At some scenarios the strategies aiming at minimizing variance showed better performance in terms of statistical measures produced.
4. The winning strategy of this paper at least in terms of highest values of IR, was the one produced during the sensitivity analysis and included the following parameters: look-back (LB) 30 days, rebalancing window (RB) 21 days, transaction cost (TC) 1% and the optimization objective max Information Ratio (IR). The combination of parameters for optimization should be studied further though, especially the combination of longer look-back period and longer rebalancing window, since their separate prolongation in the scenarios of sensitivity analysis showed the best results.
5. The transaction costs increase has a strong negative impact on the Markowitz strategy profitability. Especially, in case of IR objective the results are decreased severely. Increased TC also enhances the chance of benchmarks to drift together with the optimized portfolios, which would imply no or just a little difference in the passive and dynamic investment on cryptocurrency market.

While studying other papers producing optimized portfolios on cryptocurrencies, we have noticed that the conclusions are pretty similar between each other, but differ from the results of this research. Brauneis & Mestel (2018) made conclusions about passive investment in crypto being more profitable. Anyfantaki et al. (2018), Liu (2018), Henningsson (2019), Platanakis et al. (2018) also did not reveal any significant advantage of Markowitz optimized portfolios over the naive investment benchmarks. The clue is supposed to be in the combination of parameters or the way coins are chosen to the portfolio. The length of time range available for the research may also impact, however, mentioned papers are quite recent.

Even though this research has a complex character and addresses different aspects of investment opportunities verification for the cryptocurrency market, there is still a wide space for developing this study in several matters:

- We would like to examine the robustness of Markowitz portfolios depending on the rebalancing window extension. Since the results presented in the sensitivity analysis suggest that longer RB generated more attractive portfolio in terms of Information Ratio (IR) and annualized return (aRC). The periods under consideration are: 1 month, 2 months, 3 months.
- We have noticed that the optimization objective has also a strong impact on the strategy's performance, hence another interesting research potential would be to examine the results of switching the objective to minimize expected shortfall as a measure of risk.
- Huge drawdown shown by most Markowitz strategies suggest finding a solution for its minimizing. Hence, we would like to apply the concept of degree of financial leverage (DFL)[10]. This would imply investing not the whole wealth available but just a part of it. This would smoothen the equity line and reduce the part of string drawdowns. Interesting would be to watch the changes in Information Ratio (IR) and Maximum Drawdown (MD) measures.

References

Anyfantaki S., Arvanitis A., Topaloglou N. (2018) *Diversification, integration and cryptocurrency market.*

BAE systems (2016) *Managing the risks of cryptocurrency.*

Baek C., Elbeck M. (2015) *Bitcoin as an Investment or Speculative Vehicle? A first look.* Applied Economics Letters, vol. 22, issue 1, 30-34.

Bai Z., Li H., Liu H., Wong W. (2016) *Making Markowitz's Portfolio Optimization Theory Practically Useful.*

[10] The standard definition https://www.investopedia.com/terms/d/dfl.asp

Bariviera A., Basgall M., Hasperue W, NAiouf M. (2017) *Some stylized facts of the Bitcoin market.*

Bartos J. (2015) *Does Bitcoin Follow the Hypothesis of Efficient Market?* International Journal of Economic Sciences vol. IV, No. 2.

Baur D., Hong K., Lee, A. (2015) *Bitcoin: Currency or Asset?*

Becker F., Gurtler M., Hibbeln M., (2015) *Markowitz versus Michaud: Portfolio Optimization Strategies Reconsidered*. European Journal of Finance, volume 21, 269-291.

Bessler W., Opfer H., Wolff D., (2017) *Multi-Asset Portfolio Optimization and Out-of-Sample Performance: An Evaluation of Black-Litterman, Mean-Variance and Naïve Diversification Approaches.* The European Journal of Finance, volume 23, issue 1.

Bjordal A., Opdahl E., (2017) *Portfolio optimization in the cryptocurrency market.*

Bouri E., Gupta R., Lahiani A., Shahbaz M., *(2017) Testing for assymetric nonlinear short-and longrun relationship between bitcoin, aggregate commodity and gold prices*.

Bouri, E., Azzi, G., Dyhrberg, A. (2016) *On the return-volatility relationship in the Bitcoin market around the price crash of 2013*. (No. 2016-41) Economics Discussion Papers.

Brandvold M., Molnar P., Vagstad K., Valstad O. (2015) *Price Discovery on Bitcoin Exchanges.*Journal of International Financial Markets Institutions and Money 36, 18-35.

Brauneis A., Mestel R. (2018) *Cryptocurrency-Portfolios in a Mean-Variance Framework.*

Briere M., Oosterlinck K, Szafarz A. (2013) *Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoins.*

Briere M., Oosterlinck K., Szafarz A. (2015) *Virtual currency, tangible return: Portfolio diversification with Bitcoins*. Journal of Asset Management 16: 365–73.

Cheah E., Fry J. (2015) *Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin.* Economics Letters, vo. 130, issue C, 32-36.

Cheung A., Roca E., Su J. (2015) *Crypto-currency bubbles: an application of the Phillips-Shi-Yu (2013) methodologyon Mt. Gox Bitcoin prices.* Applied Economics, vol. 47, issue 23, 2348-2358, 45

Ciaian, P., Rajcaniova A., Kancs D. (2016) *The economics of Bitcoin price formation*. Applied Economics, 48 (19), 1799-1815.

Corbet S., Lucey B., Yarovaya L. (2018) *Datestamping the bitcoin and ethereum bubbles.* Finance Research Letters 26: 81–88.

Dyhrberg A. (2015a): *Hedging capabilities of bitcoin. Is it the virtual gold?* In: Finance Research Letters, Volume 16: P. 139-144.

Dyhrberg A. (2015b) *Bitcoin, gold and the dollar – A GARCH volatility analysis.* Finance Research Letters Volume 16, 85-92.

EBA 2014, *EBA Opinion on virtual currencies.*

Eisl, A., Gasser, S. M., Weinmayer, K. (2015). *Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?*

Elendner H., Trimborn S., Ong B., Lee T. (2016) *The Cross-Section of Crypto-Currencies as Financial Assets: An Overview.*

Engle R. (2002) Dynamic Conditional Correlation. Journal of Business & Economics Statistics, vol. 20, issue 3.

Fabozzi F., Gupta F., Markowitz H. (2002) *The legacy of Modern Portfolio Theory*. The Journal of Investing 11(3) 7-22.

Garcia D., Tessone C., Mavrodiev P., Perony N. (2014) *The digital traces of bubbles: feedback cycles between socio-economic signals in the Bitcoin economy*. Journal of the Royal Society.

Glaser F., Zimmermann K., Haferkorn M, Weber. M., and Siering M. (2014) *Bitcoin - Asset or Currency? Revealing Users' Hidden Intentions.*

Henningsson M. (2019) *Can cryptocurrencies enhance portfolio performance?*

Ivanova M., Dospatliev L. (2017) *Application of Markowitz portfolio Optimization on Bulgarian Stock Market from 2013 to 2016.* International Journal of Pure and Applied Mathematics, volume 117 (2) 291-307.

JI Q., Bouri E., Gupta R., Roubaud D. (2017) *Network Causality Structures among Bitcoin and other Financial Assets:A Directed Acyclic Graph Approach.*

Keller W. (2014) *Momentum, Markowitz, and Smart Beta: A Tactical, Analytical and Practical look at Modern Portfolio Theory.*

Keller W., Butler A., Kipnis I., (2015) *Momentum and Markowitz: A Golden Combination.*

Klabbers A. (2017) *Bitcoin as an investment asset: The added value of bitcoin in a global market portfolio.*

Kosc K., Sakowski P., Slepaczuk R. (2019*) Momentum and contrarian effects on the cryptocurrency market.* Physica A: Statistical Mechanics and its Application, vol. 523, 691-701. 46

Kristoufek L. (2013) *Bitcoin meets Google Trends and Wikipedia: Quantifying the relationship between phenomena of the Internet era*. Scientific Reports 3:3415.

Lee D., Guo L., Wang Y. (2017) *Cryptocurrency: A New Investment Opportunity?*

Levy H., Markowitz H. (1979) *Approximating Expected Utility by a Function of Mean and Variance.* American Economic Review 69(3): 308-17.

Liu Y., Tsyvinski A. (2018) *Risks and Returns of Cryptocurrency.*

Magro, P. (2016) *What Greece can learn from bitcoin adoption in Latin America.* International Business Times.

Markowitz, H. (1952) *Portfolio Selection*. The Journal of Finance, 7(1): 77–91.

Nadarajah S., Chu J. (2017) *On the inefficiency of Bitcoin.* Economics Letters, vol. 150, issue C, 6-9.

Nakamoto S., (2008) *Bitcoin: A Peer-to-Peer Electronic Cash System.* Cambridge: MIT Press.

Platanakis E., Sutcliffe C., Urquhart A. (2018) *Optimal vs naïve diversification in cryptocurrencie*s. Economic Letters 171.

Polasik M., Piotrowska A., Wisniewski T., Kotkowski R. (2015) *Price Fluctuations and the Use of Bitcoin: An Empirical Inquiry*. International Journal of Electronic Commerce 20(1):9-49.

Sakowski P., Slepaczuk R., Wywial M., (2016) *Can we invest on the basis of equity risk premia and risk factors from multi-factor models?* Economics and Business Review volume 2(16).

Tiwari, A., Jana R. K., Das D., and Roubaud D. (2018) *Informational efficiency of Bitcoin—An extension.* Economics Letters 163: 106–9.

Trimborn S., Li M., Hardle W., (2017) *Investing with Cryptocurrencies – A liquidity Constrained Investment Approach.*

Yelowitz A., Wilson M. (2015) *Characteristics of Bitcoin users: analysis of Google search data.* Applied Economics Letters, 22:13, 1030-1036.

Yermack D. (2014) *Is Bitcoin a Real Currency? An Economic Appraisal*.

https://www.investopedia.com/ask/answers/010815/what-difference-between-sharpe-ratio-andinformation-ratio.asp

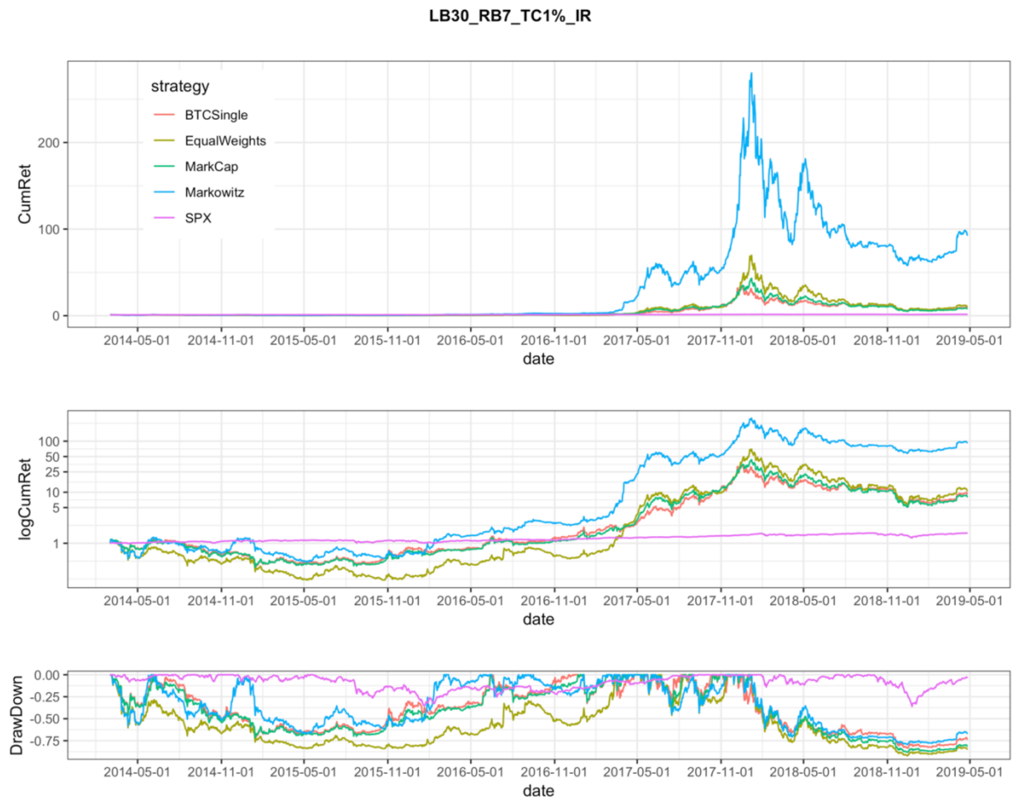
https://www.investopedia.com/terms/m/maximum-drawdown-mdd.asp

https://stooq.pl

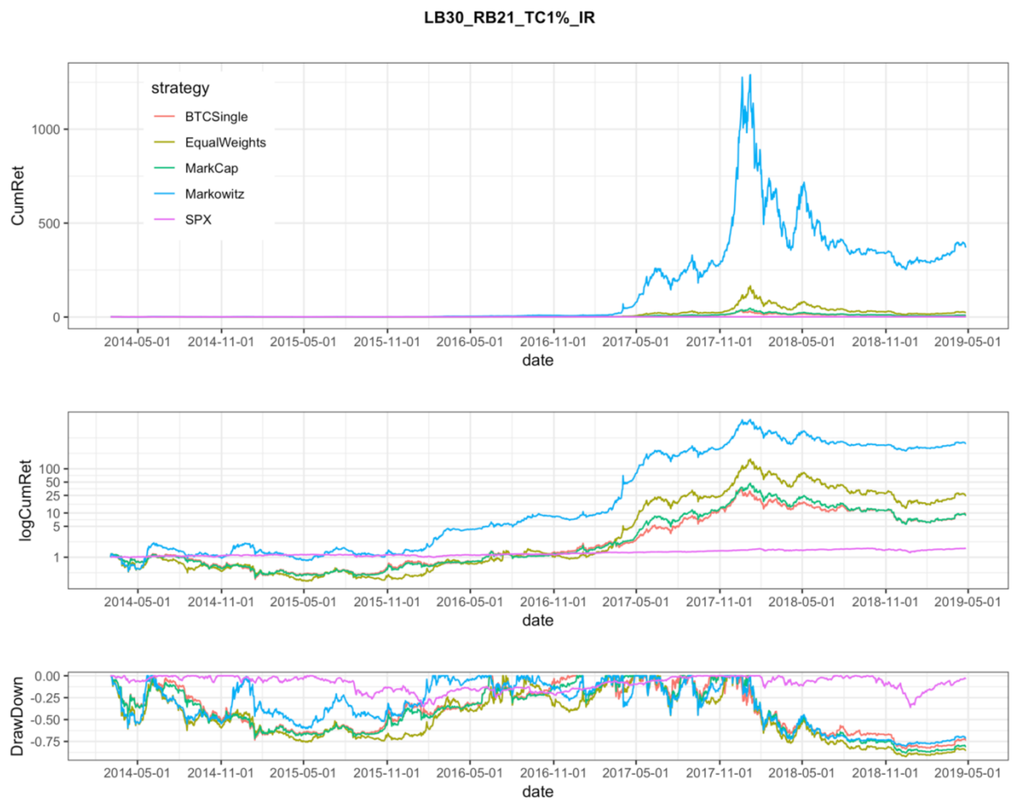
https://www.investopedia.com/terms/d/dfl.asp

Appendix 1. Equity lines for the sensitivity analysis strategies (see Figure 11 for naming convention)

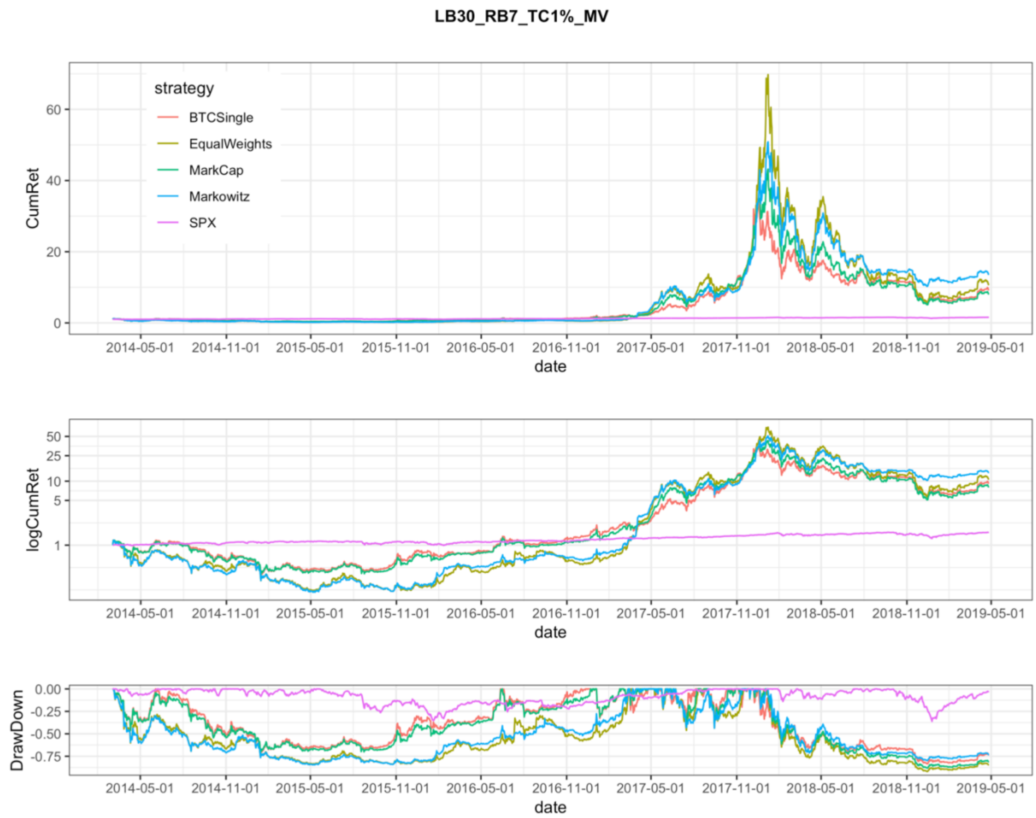
(1) LB30_RB7_TC1_IR



(2) LB30_RB21_TC1_IR



(3) LB30_RB7_TC1_MV



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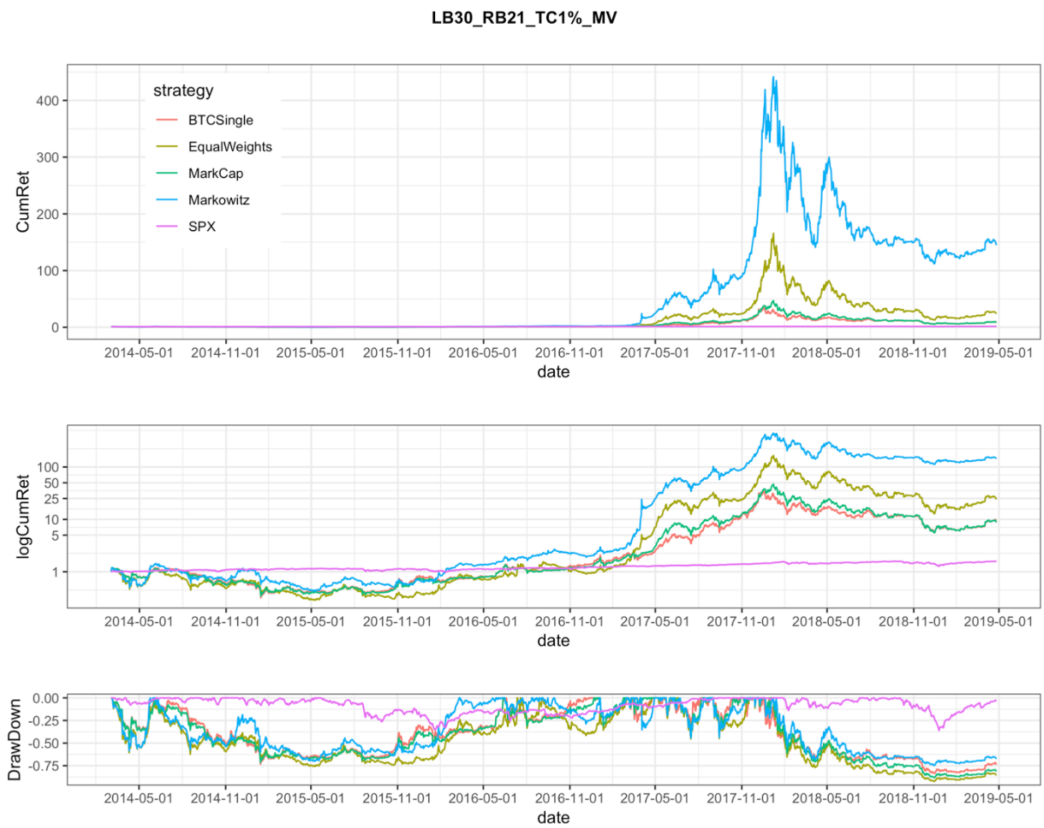
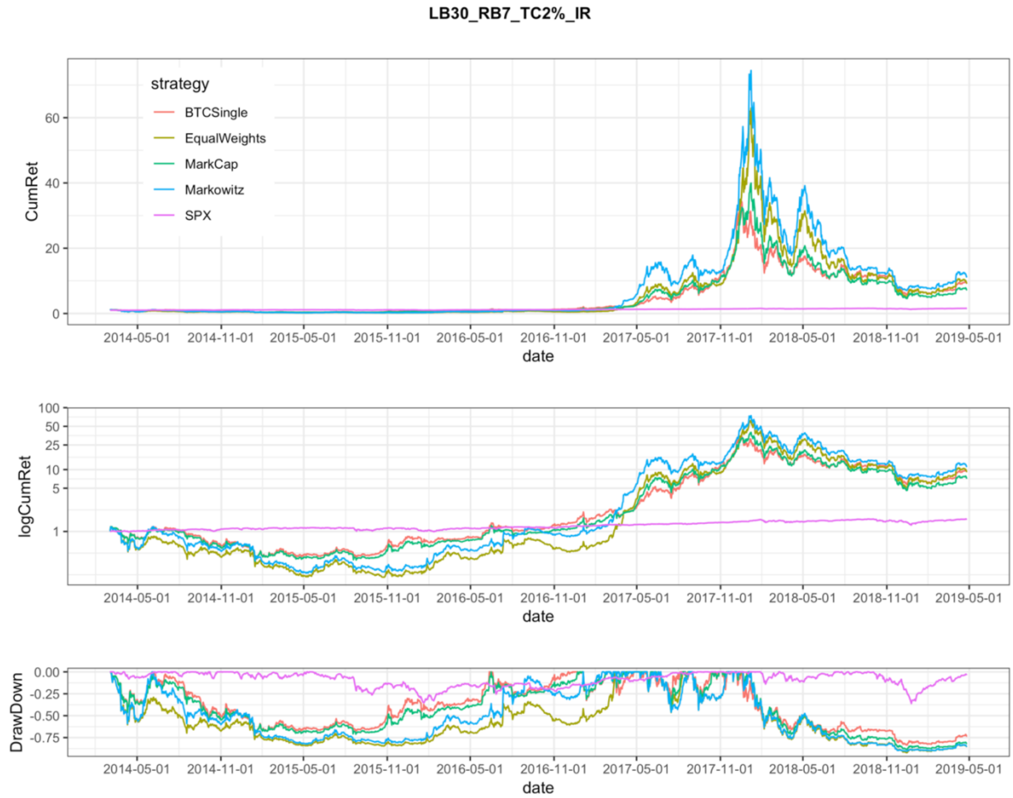


(5) LB30_RB7_TC2_IR



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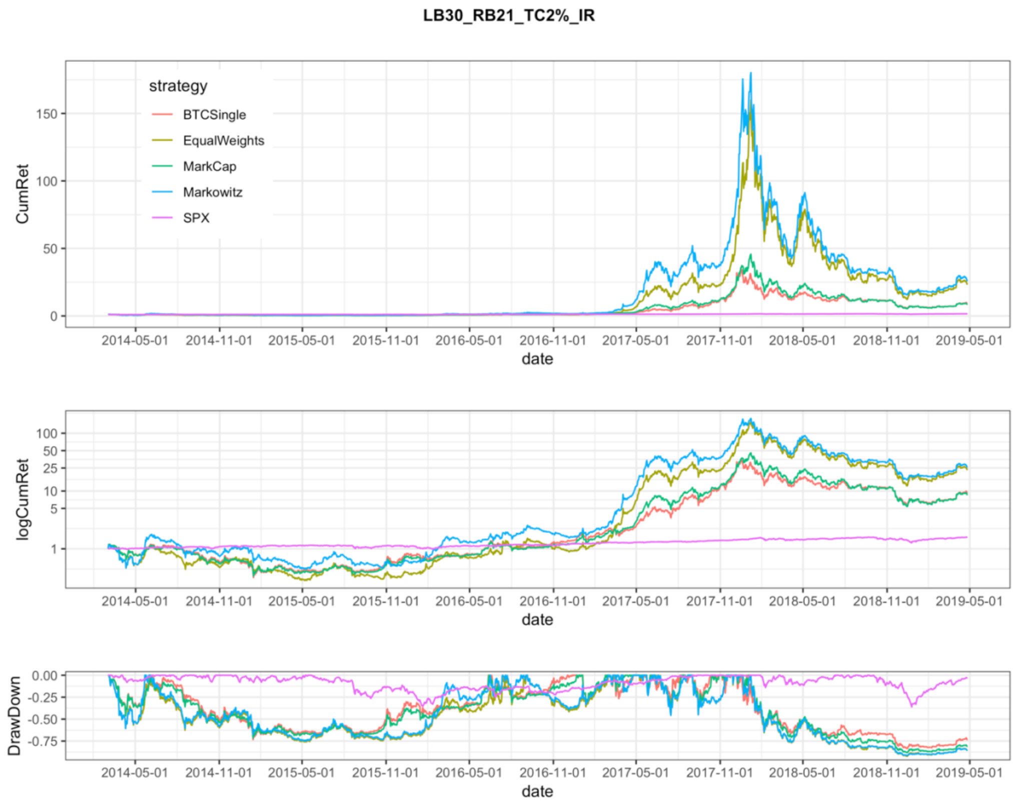
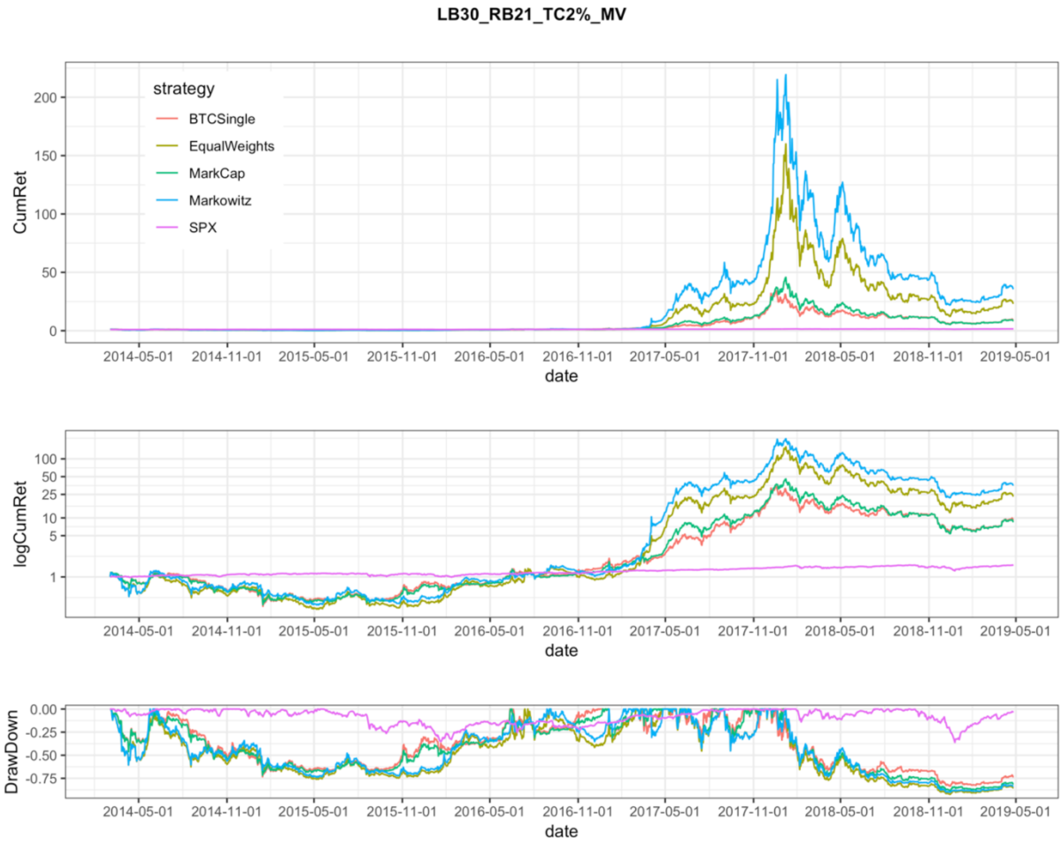
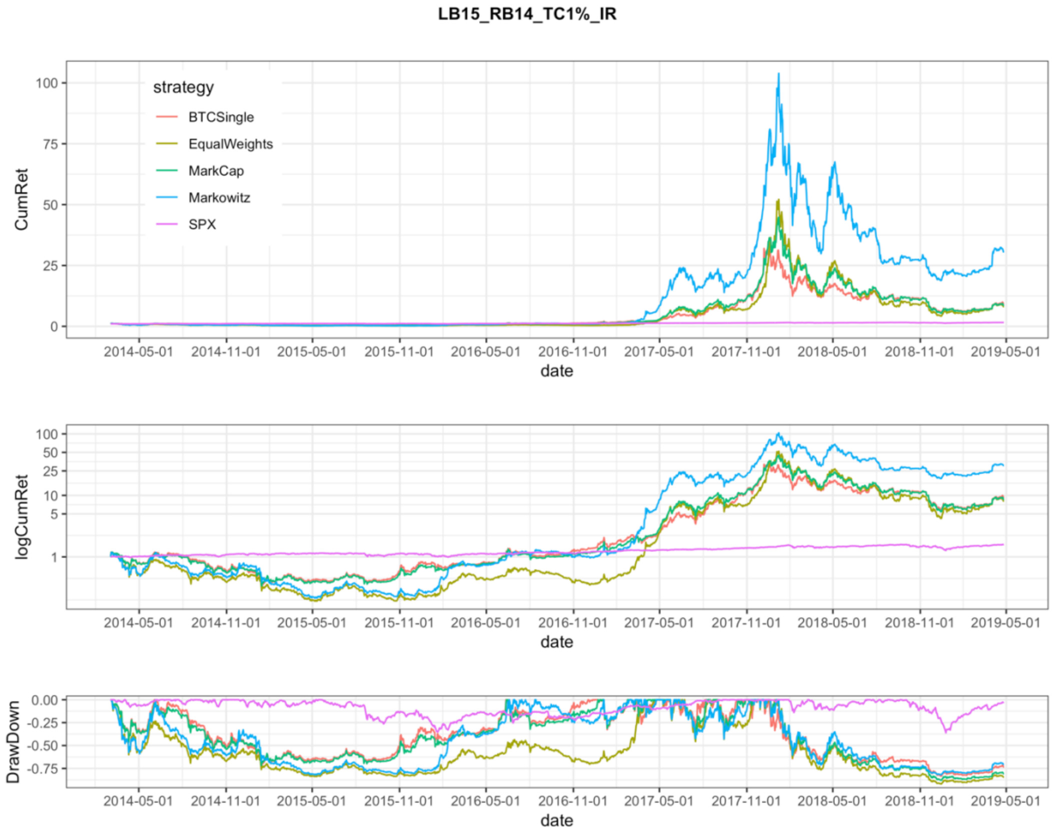


(7) LB30_RB7_TC2_MV

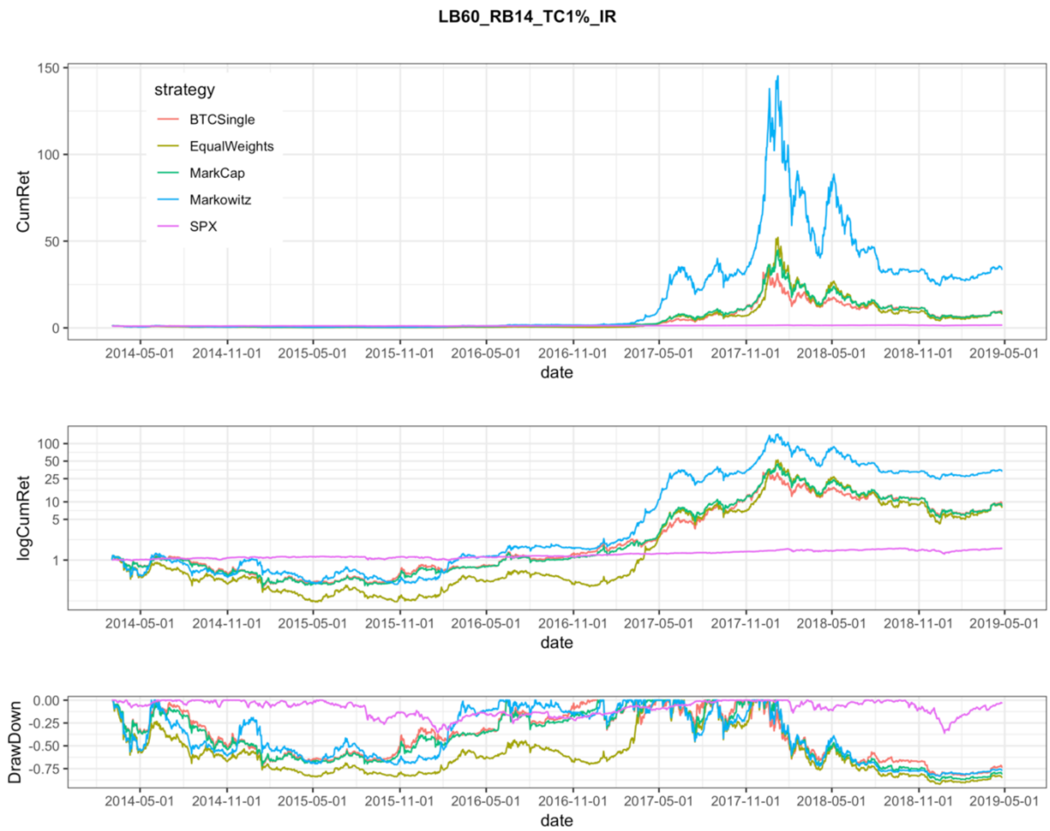
(8) LB30_RB21_TC2_MV



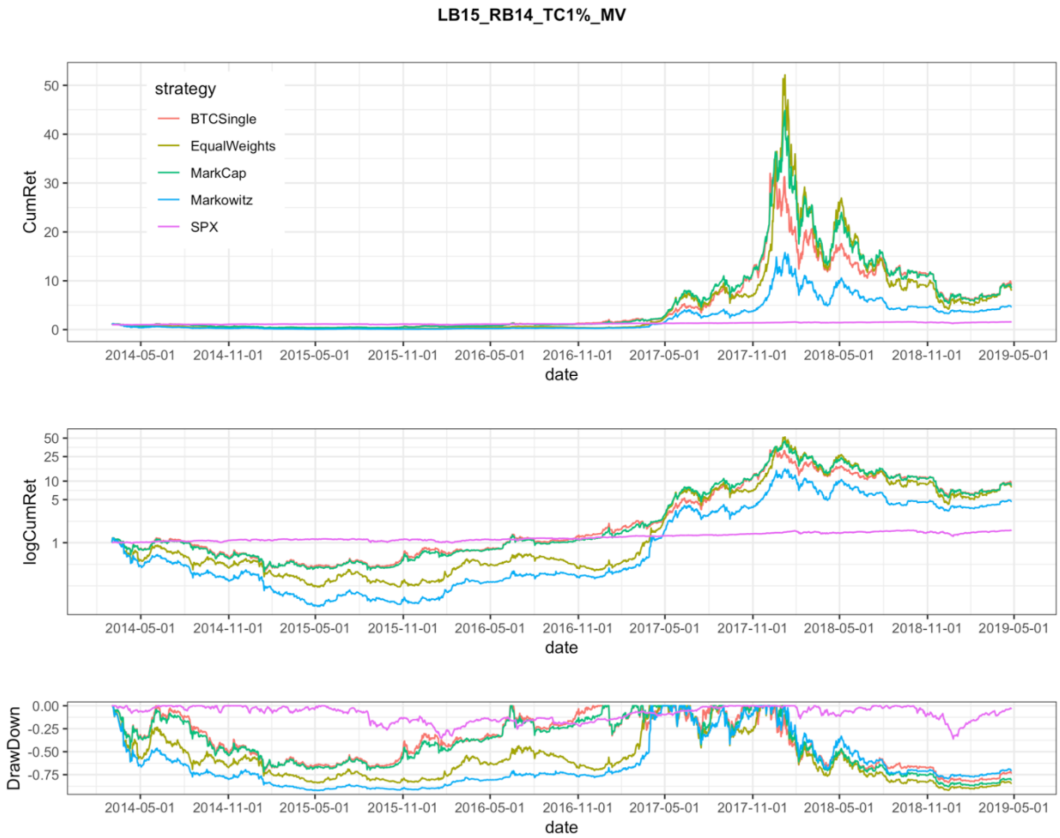
(9) LB15_RB14_TC1_IR



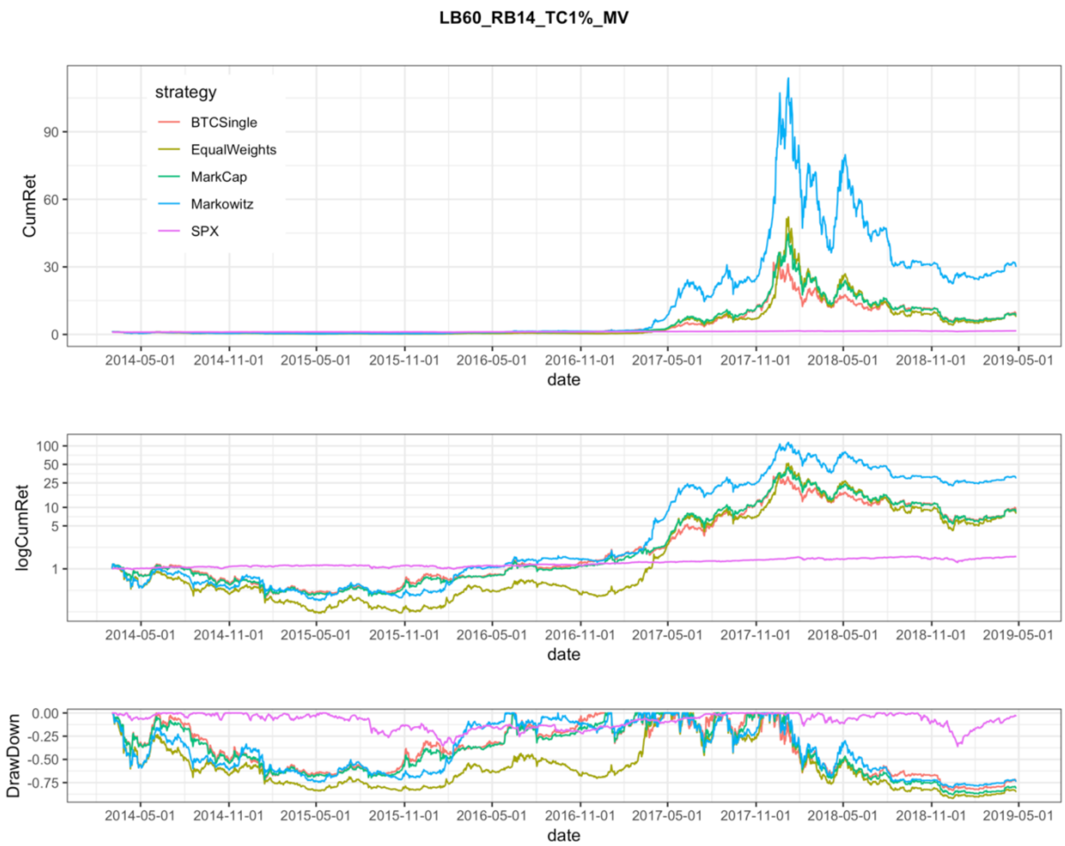
(10) LB60_RB14_TC1_IR



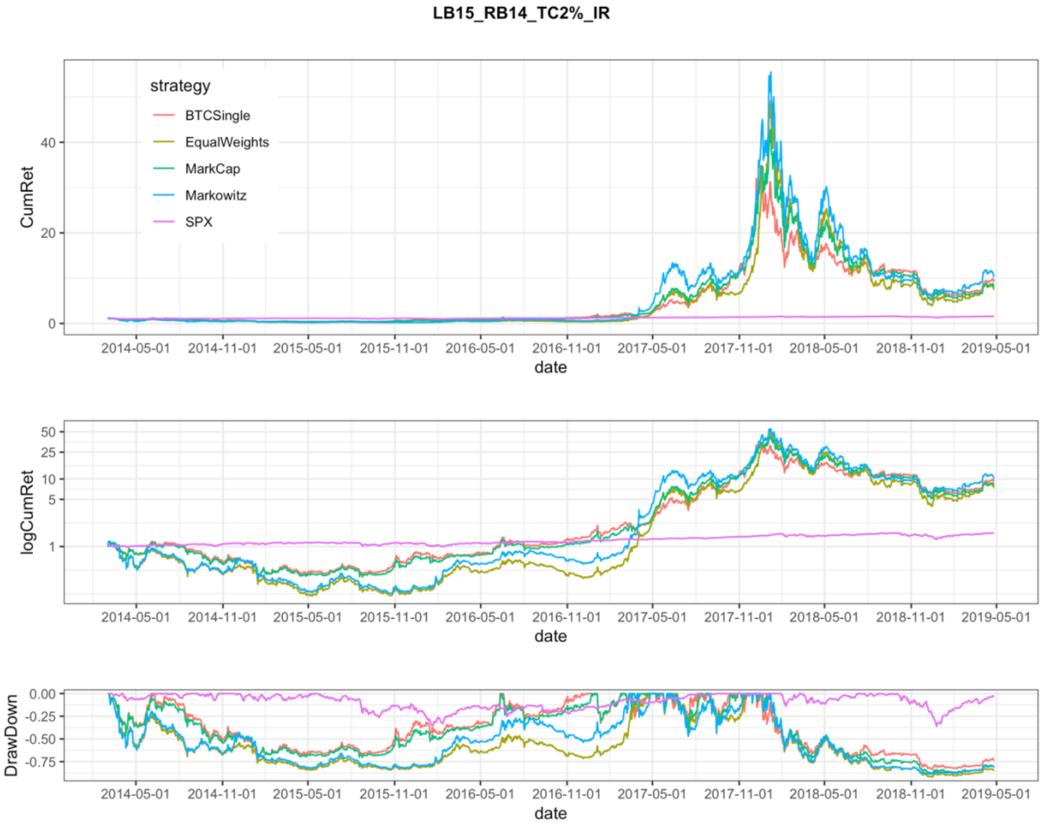
(11) LB15_RB14_TC1_MV



(12) LB60_RB14_TC1_MV

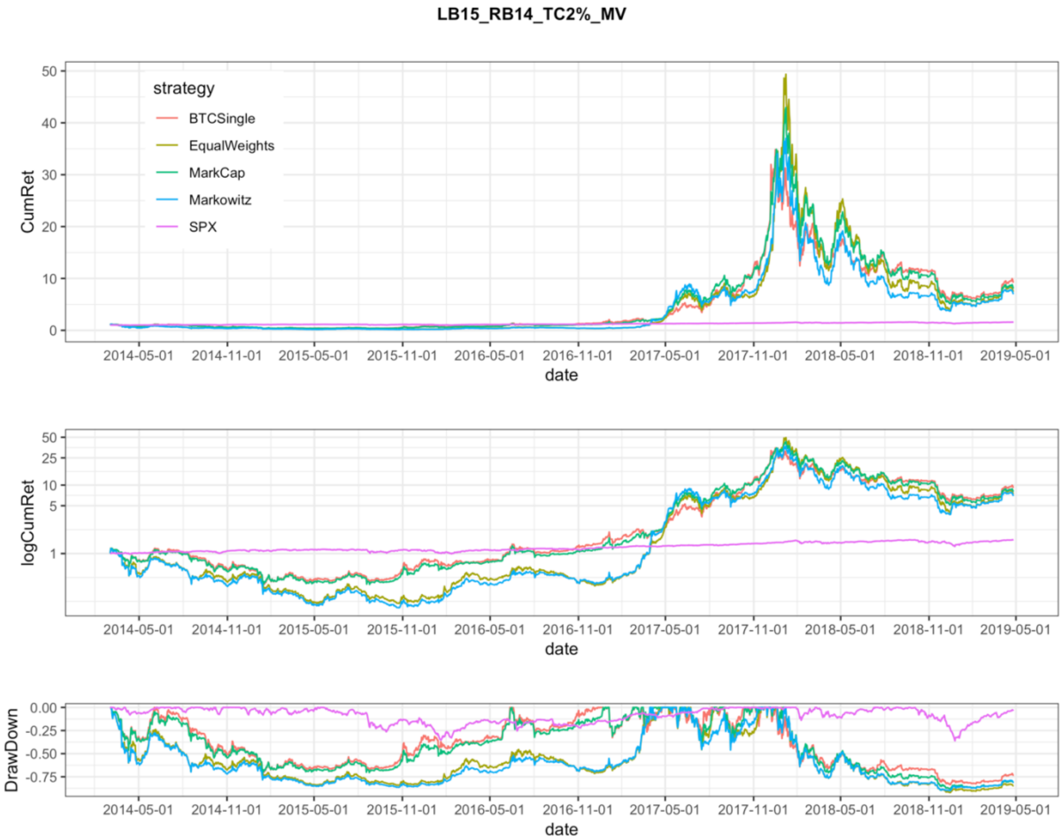


(13) LB15_RB14_TC2_IR

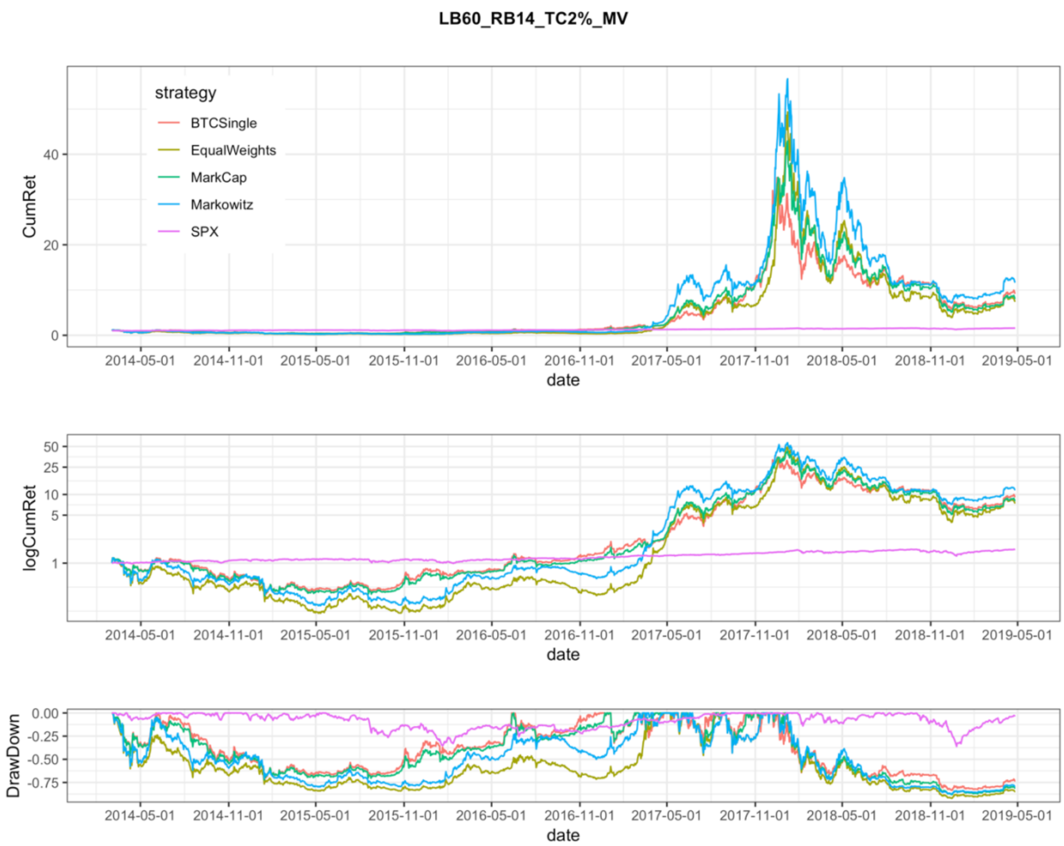


(14) LB60_RB14_TC2_IR

(15) LB15_RB14_TC2_MV



(16) LB60_RB14_TC2_MV



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